Part 1

Kießwetter/Zimmermann, Madrid, Nov. 1998

Information about the Hamburg Model to foster mathematically talented pupils

Chronology, Organization, Statistics, Results from Interviews etc.

1981 (February) - Contract about cooperation between a working group at the Institute of Psychology II at Hamburg University and the CTY at Johns-Hopkins-University in Baltimore

1983 until 1986 - research project “Identification and Fostering of Mathematically Gifted Students” (Hamburg Model) sponsored mainly by a grant from the Ministry of Science

1985 - Foundation of the William-Stern-Society for Research about and Advancement of talented youth, which is in charge of the Hamburg Model since 1986

Statistics etc.

3 classes for the middle grades (one in grade 7, 8 and 9; average age 13 to 15), each of them with students of similar age, one group for the upper grades (grade 10 to 13, average age 16 to 19) - in each group are 30 - 45 pupils.

The groups come together on Saturday between 9.30 and 12.30 am. at the Department of Education of Hamburg University, which is located at the center of Hamburg with good traffic connections. They meet 20 to 24 times in a year.

Average attendance in classes from 1983 up to 1989 (there are hardly any changes until now):

At the beginning of year number \( x = 2 \ 3 \ 4 \ 5 \ 6 \)

from all students of a new group still \( y = \) 85% 72% 57% 44% 20%
stayed in our fostering project.

Through all years, we have a constant rate of some 30% girls in our classes.

The parents help to run our project by donations since 1986. Furthermore, there is a lot of work without payment. The amount of money necessary for one year and one student in the middle grades is some 480 DM (corresponding to 390.000 Ptas. per year, ca. 32500 per month and 16000 per session). We ask the parents to pay an amount which corresponds to their possibilities. So there are no economic criteria for participation. Until now the parents helped us to run the courses without difficulties.

To select the participants requires a lot of time and different stages:

- At the beginning of every year mathematics teachers of all sixth grades (11-12 years old pupils) at schools (Gymnasium, comprehensive schools) in Hamburg get information about our project. They are asked to contact parents with children, whom they assume to be mathematically talented.

- Parents who are interested in our program can contact us to get further information.

- We send an information brochure to the parents and their child where we give special material about the SATM (including an example of a specific form of the SATM). This American test has been developed for high school students (age 16 or older) and as college entrance examination test. Some test items require additional knowledge from the middle grades. The pupils have one month to prepare for the test by this material.

- After one month of preparation most of the children apply for the test and join the procedure (some 130 to 200 students (Hamburg has approx. 1.7 million inhabitants). These students work on two tests on a Saturday morning at the end of their sixth grade: for one hour they work on 60 multiple-choice-problems of German translation of the SAT-M and - after a break - for two hours on 7 open-ended problems of the HTMB.

- The results of the tests are nearly constant over the years. There is an offer to 40 to 50 children to join our program. Nearly all of them accept our offer and become participants of our program.

Observations and results of interviews

- In our first three groups we administered an IQ-test after the end of the selection-procedure, too. Result: The pupils had an average IQ of 145 (some test scores were higher than 160, the IQ of a handicapped pupil (language deficiencies) was less than 100).

- The participants of our program are multi-talented, so they are very often gifted not only in the domain of mathematics. Many are playing an instrument in a very good way. Their ability to speak foreign languages is very often above average, too.
- If we compare girls and boys of our program, it seems to be obvious that girls prefer group work and play more often an instrument. Boys work or play with a computer much more often and it is nearly a matter of fact that they play chess.

- Our pupils demonstrated their very broad range of interests when invited scientists - normally university professors - lectured about large prime numbers, development of mathematical models in medicine or especially about imbedding mathematics into history and culture.

- Visitors were again and again impressed by the fact that our pupils were used to a very effective way of communication using much less words and more verbal “chunks” than normal students.

- Several intensive psychological interviews had been conducted with participants of our project. The respective results were published in a thesis. The main reason for the major interest in our project is summarized in this book as follows: “It is very important that there is the possibility to contact other pupils of similar talent and interests as well as working on challenging mathematical problems.”

There are further hints, that the possibility to compensate loneliness caused by unusual interests by joining our program might be as important as the mere interest in mathematics. This has to be respected. Therefore, our tutors have order not to disturb talks about subjects, which are not directly related to the problem field, to be discussed at that morning.

Selection of the teachers of our program

We select teachers by using the following criteria:

- sufficient experience and success in creative production of new mathematics at least in the domain of elementary mathematics,

- Ability for and experience in observing and interpreting episodes in creative mathematical thought processes - in his or her own person (by introspection realizing own deficiencies and making transfer from this fact) and within other persons. When our students work on our problems, there are hardly any thought processes which are completely wrong. The tutor has to realize this, get the positive aspects of the thoughts as quick as possible, to react in an appropriate way.

- Humanistic qualities (to keep back, to have no image problems, willing to help other people...)

To put it all together:

It is by no means sufficient, to have a good University examination and some years of experience at school to meet the needs and the rich ideas of highly talented pupils and to guaranty to them an optimal environment to “flourish” in a self-determined way with or in open problem fields.
1. What is “high mathematical giftedness”?

Considering the high complexity of the subject on one hand and such a general question on the other, it is obvious, that one cannot expect a well-defined, precise and general answer to this question. Furthermore, it cannot be independent of contemporary and cultural influences.

In addition, promotion projects are always forced to follow a pragmatic approach. An evaluation of a student as “mathematically highly talented” is limited by the used testing procedure. It cannot be defined by a single method.

Therefore, our first question has to be modified to:

2. How to identify mathematically highly talented students?

2.1 Main principles for a conclusive test

When we tried to develop an appropriate test for identification of mathematically talented students, the following criteria had to be met:

It should assist in achieving a reliable forecast with a high degree of likelihood for the present and future in respect of:

a) the requested cooperation as a member of the team of pupils of the talent project (which, besides other aspects, focusses on the simulation of research situations and processes) and

b). outstanding mathematical achievements as an adult.

In both cases the support by a suitable environment is very important.

2.2 Principal constraints and boundary conditions - on measurability and validity

There might be an analogy to Heisenberg’s uncertainty principle in respect of possible procedures of measurement and outcome of tests. There are also pairs of variables in tests to identify mathematically talented students, which are “canonically conjugated”: neither objectivity nor
validity of the test can be “measured” with the same accuracy at the same time. The more we meet one criterion, the less we meet the other one and vice versa.

There are even components of mathematical talent/giftedness, which cannot be measured by a test at all. E. g. aspects which only can be observed after a person emerges himself in a problem for weeks or months of trial and error, different approaches and experience of failures. Very often achievements are the result of time consuming efforts of coping with errors in an appropriate way.

2.3 Psychological and epistemological constraints for the construction of our test HTMB

(Hamburger Test für mathematische Begabung; Hamburg Test for Mathematically Talented Pupils)

- 12 years old pupils command only a very limited mathematical knowledge.

- Their perseverance should be tested but not unduly to prevent restricted mathematical thinking and problem solving.

- As a consequence we can expect only a restricted catalogue of relevant mathematical thought-activities (categories).

- The quality of these thought-activities is related to our mathematical belief-system.

- Our mathematical belief-system provokes open-ended problems, which require a considerable amount of time for problem solving, therefore, we can present only a limited number of problems to the pupils.

- The problems of the tests should allow various approaches by pupils to account for different profiles of mathematical talent.

2.4 Procedure of constructing the HTMB by some kind of Delphi-method

Even though the process of convergence was mainly related to Karl Kießwetter and led to 6 categories and finally to the HTMB it also might be interpreted as the result of a modified Delphi method as used by commercial enterprises.

There was a phase of preparation, including discussions with mathematicians, teachers etc. as well as an intensive study of relevant literature. Stimulating answers in imaginary discussions were
especially found in the books of Pólya and in resources from the history of mathematics. On this background a set of items for the HTMB was developed.

Based on this background a number of items were designed for the HTMB.

Finally, the fine tuning of the HTMB was done in accordance with the Delphi method by a group of competent mathematicians, mathematics educators and mathematically versed teachers. The HTMB was tested in Baltimore with pupils of age 12 and in Hamburg with pupils of age 16, before it was administered to the first applicants of our courses.

**Concluding remarks:**

Meanwhile for the past 15 years an annual group of some 40 to 45 pupils was selected (in all almost 700 pupils) by using our test. Our preliminary conclusion is: the testing procedure is “sufficient” in the sense that there is a high degree of likelihood of the selected pupils to be successful members of their workgroups. Furthermore, there are many examples of pupils who did well in the HTMB to also produce the best ideas. We are unfortunately faced with the predicament of not knowing in how far the HTMB test itself is “necessary” for the later success of tested pupils. Perhaps there might be excellent pupils who are not successful in this test. Because of this possibility, we also included a little corrective measure in the test to take care of students who during the tests were too excited, too slow or otherwise hindered to perform well by allowing them to sent us their ideas by mail the next day.
Part III : Rationale of the Testing Procedure

**Connectedness of influencing factors/Complexity of the situation**

**mathematical belief system:**
- professional work and achievements of mathematicians
  - mathematics is **theory building**
  - **Hamburg Model:** simulation of research processes in elementary mathematics

**pragmatic boundary conditions:**
- testing-situation (limits of time)
- profiles of mathematical giftedness
due to age: (12 years)
- restricted knowledgebase

**test**

**6 categories of complex mathematical thinking processes**
- **H T M B**
“There is a strong need for naivete to be creative. I have the strong impression that - normally - there is placed to less emphasis on creativity in university teaching. The main focus is on the transmission of the subject matter, i. e., of knowledge. In contrast to this tradition there is nearly no interest in playing with the subject matter.

So, it seems to me, as if many professors take it as a waste of time or think it to be childish to cope with the subject matter by playing with it. As to their opinion one can only start to think about the possibility of creative processes, after the subject matter is completely mastered.

Creativity is the ability to restructure given information in a fruitful way and to augment it.”

Quotation from

**Gerd Binnig** (Physics-Nobel-laureate 1986):**

**Aus dem Nichts - Über Kreativität von Natur und Mensch.**

*(Out of Nothing - on Creativity of Nature and Men) (Piper, München, 1989)*
Starting points for the development of “problem-fields” and the construction of small mathematical theories with highly talented pupils

1) Cross-out-squares

\[
\begin{array}{cccc}
-7 & (6) & 5 & 9 \\
5 & 4 & 3 & 7 \\
3 & 2 & 1 & 5 \\
6 & 5 & 4 & 8 \\
\end{array}
\]

Procedure:

Chose a number (in our example we take 6 first) and cross out all the numbers which are in the same row or in the same column. Then take another number which is not crossed out and cross out all 

\[
\begin{array}{cccc}
 & & & \\
 & x & x & (3) \\
 & & & \\
(3) & x & x & x \\
& & & \\
& x & x & (8) \\
\end{array}
\]

Definition:

A square of numbers is called a cross-out-square if each procedure yields the same sum.

2) Dominoes

3) Magic squares

4) Boß-puzzle

5) Viviani’s Theorem
6) In our time we use still numbers like 12, 24, 60, 90, 144, 180, 360 for certain graduations (time, angle - our name for this numbers is “scale-numbers”) and not the numbers 7, 10, 25, 100, 300, 1000.

7) Nontransitive Valuations

7 persons of a selection committee have to make ratings about the candidates A, B and C, 3 points can be given for the best one, 2 for the second and so on. The distribution of the points across the committee is given by the following table:

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<tr>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tr>
<td>A</td>
<td>3</td>
<td>1</td>
<td>1</td>
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<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
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The table indicates:

Every candidate gets 14 point, but if there are consecutive decisions between pairs of candidates, the table indicates, that A is rated better than B (4:3 votes), B is rated better than C (4:3) but - surprisingly - not A is better than C but C is rated better than A (4:3)

Associations: A > B > C > A ?? ; dice games (Efron-dices)
# On different Profiles of Mathematical Talent

<table>
<thead>
<tr>
<th></th>
<th>Claus</th>
<th>Dieter</th>
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<tbody>
<tr>
<td>achievement in tests</td>
<td>best in HTMB</td>
<td>best in SATM</td>
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<tr>
<td></td>
<td>did not work on 14 of the items of the SAT</td>
<td>worked on all items of the SAT except one</td>
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<td></td>
<td>better in finding of patterns, rules and</td>
<td></td>
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<td></td>
<td>new problems in HTMB</td>
<td></td>
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<tr>
<td>cognitive style</td>
<td>Intuitive</td>
<td>analytical/formal</td>
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<tr>
<td>quality of work</td>
<td>many very good and extraordinary ideas</td>
<td>very good verbal expressions, good in formalizing,</td>
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<td></td>
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<td>original proving,</td>
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<tr>
<td>system of values</td>
<td>more childish (some social difficulties at</td>
<td>very interested in proving and exact arguments</td>
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<td>the beginning)</td>
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<tr>
<td>IQ-Test</td>
<td>150 good in the verbal domain, analogies,</td>
<td>150 good in computation</td>
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<td>associative knowledge</td>
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