
Chemical Organizations in a Toy Model of the Political System

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Since sociology is seeking mechanisms for explaining social phenomena, we discuss in this paper, whether and how the metaphor of a chemical reaction network can be employed as a formal mechanism to describe social and political systems. A reaction network is a quite general concept, which allows to model a variety of dynamical systems. Further more, a set of powerful tools can infer potential dynamical properties from the networks structure. Using a toy model of the political system inspired by Luhmann, we demonstrate how chemical organization theory can be applied and can give insight into the structure and dynamics of the resulting model. Chemical organization theory allows identifying an overlapping hierarchy of important sub-systems in these networks.

1 Introduction

Sociology is seeking mechanisms that explain social phenomena [1]. Mechanisms can be described in various ways, ranging from rich linguistic descriptions to precise mathematical formulas [2]. In general it is desirable to obtain a formal description of a social mechanism. However, there is the danger to lose important details of the social phenomenon during the process of abstraction [3] or to arrive at a more complex model than actually required to explain the designated phenomenon. Therefore it is necessary to possess a rich toolbox, which offers different mechanisms for different levels of abstraction.

Here we discuss one specific but quite powerful metaphor, the chemical reaction network, which can be used to describe a large variety of *dynamical* phenomena from fields of study like economy and sociology. Furthermore, we describe chemical organization theory [4, 5], which allows identifying an

overlapping hierarchy of important sub-systems in these networks. These sub-systems are closely connected to dynamics, because they are candidates for self-sustaining (maybe even autopoietic) systems [6, 7].

Note that our approach is different from classical social network analysis (SNA) [8] and especially from SNA using Galois lattices [9, 10, 11], which aim at identifying structures in networks, but which do not rigorously relate these structures to (potential) dynamics. Another important difference is that in SNA, networks represent similarity relationships, for example, between actors or between actors and events these actors participate in. However, in our approach, a network consists of production rules, which have a clear temporal interpretation and direction. Our network defines the structure of a dynamical model. Therefore, the hierarchies that we detect are fundamentally different from those arising in SNA [8] or formal concept analysis [10]. In our case, an element of the hierarchy represents a possible relatively stable state of the system modeled by the reaction network. The dynamics appears as a movement within that hierarchy, and not as a structural change of the hierarchical structure itself [11], although both views are related and could be integrated.

The techniques described here are not only useful to model social systems. In the same way as we build social models, we can use these techniques to build and analyze multi-agent systems (MAS), which is the fundamental philosophy of Socionics [12]

2 Chemical Organization Theory

Chemical organization theory deals with reaction systems. The basic idea of the theory is to identify hierarchical structures in terms of chemical organizations, which are closed and self-maintaining sets of elements. These elements can be chemical species, political decisions, social communications, or economical products. Organizations can be visualized using a Hasse-diagram (as shown in Fig. 1, right), which provides a powerful graphical representation of the network’s inherent structure. The theory has been inspired by Fontana’s and Buss’ notion of a biological organization [13].

The networks investigated in this study are equivalent to catalytic flow systems, which are reactions systems where all molecules act only as catalysts and where all molecules decay at a positive rate. Therefore, we require only a simplified version of the chemical organization theory [13]. In particular, we can exclude mass conservation and thus can exclude a more general concept of self-maintenance ([5], Def. 10). The required theory can be explained in a quite compact way. In order to explain the theory, we will stick to its terminology. So, we call the elements of a system *molecules* and the relation among elements *reactions*. Molecules and reactions together form a reaction network as exemplified by Fig. 1, left. Let us first define clearly, how the systems look like that we will analyze. We call these systems catalytic networks. The term “catalytic” is used, since all reactions are assumed to be fully catalytic.

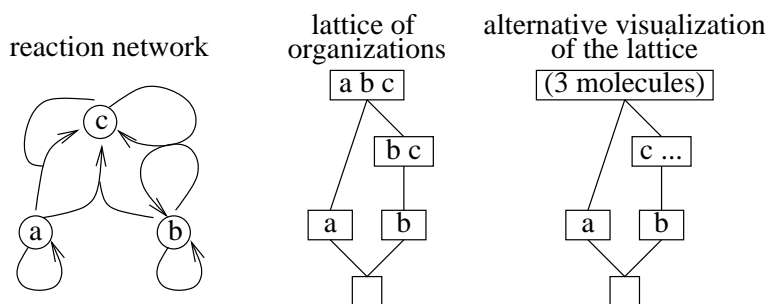


Fig. 1. A catalytic network (left) and its lattice of organizations (right). An arrow denotes a catalytic reaction where all educts are catalysts. In a catalytic network all molecules decay, which is not depicted.

A catalytic network is a set of molecules together with a set of reaction rules. A reaction rule is just a pair of sets of molecules. A reaction consists of a left hand side and a right hand side. A reaction is interpreted in the following way: The left hand side denotes a set of molecules that are sufficient to produce the molecules on the right hand side. Furthermore, if the molecules of the left hand side are present in a reactor, then all molecules on the right hand side are produced by this reaction (at least at a low rate). Since we assume a catalytic flow system, the molecules on the left hand side are not used up when the molecules on the right hand side are produced. Molecules vanish only through a general dilution flow, which is equivalent to a spontaneous decay. In the political system we analyze here, a molecule like a denotes a decision. According to Luhmann, a (political) decision is a special communication [14]. Dilution models the vanishing influence of a decision over time.

2.1 Chemical Organization Theory for Catalytic Flow Systems

In this section, we describe the basic concepts of a simplified version of the chemical organization theory, which is sufficient for political systems investigated here and other social systems [15]. The simplified version is limited to so called catalytic flow systems, which are reaction systems where all molecules act always as catalysts and all molecules decay spontaneously [5]. Despite of these limiting assumptions, the simplified version of the theory possesses already a wide application range, as exemplified by our investigations of the political system and economic networks [15]. Furthermore, various systems in Biology and related disciplines can be treated with the simplified theory, cf., hypercycle [16], replicator equation [17], autocatalytic networks [18].

The reaction system is given as a catalytic network $(\mathcal{M}, \mathcal{R})$, where \mathcal{M} is a set of molecules and \mathcal{R} is a set of reactions of the form $(A \Rightarrow B) \in \mathcal{R}$, where A and B are sets of molecules, $A, B \subseteq \mathcal{M}$. For an example see Fig. 1, left.

The set of reaction rules \mathcal{R} contains all reaction rules of the model. A reaction rule possesses the form $A \Rightarrow B$, where A is the set of molecules on

the left hand side and B is the set of molecules on the right hand side. So, the reaction rule $a+b \Rightarrow c$ can be written as $A \Rightarrow B$ with $A = \{a, b\}$ and $B = \{c\}$. Note that here it is sufficient that A and B are sets and not multisets, because we deal solely with catalytic flow systems.

Example (catalytic network): The pair $(\mathcal{M}, \mathcal{R})$ is a catalytic network with the set of molecules $\mathcal{M} = \{a, b, c\}$ and the set of reaction rules $\mathcal{R} = \{a+b \Rightarrow c, a \Rightarrow a, a+c \Rightarrow c, b+c \Rightarrow b+c, b \Rightarrow b\}$ (Fig. 1). The double arrow \Rightarrow denotes a catalytic reaction, ie. $A \Rightarrow B$ is equivalent to the chemical reaction $A \rightarrow A+B$.

In order to illustrate the meaning of this reaction system, we give an example of a dynamical process that is governed by such a network: Imagine that the process takes place inside a reaction vessel or reactor. Assume that the reactor contains instances of the molecules, such that the concentrations of the molecules are: $[a] = 20$, $[b] = 5$, $[c] = 4$. This concentration vector can be interpreted in the following way: there are 20 units of a , 6 units of b , and 4 units of c present in the reactor. The reactor represents an instance of our reaction system (catalytic network). Now we can simulate the dynamics by applying the reaction rules to the reactor. E.g., we can apply rule $a + b \Rightarrow c$, which increases the amount of c by one. Note that it does not decrease a nor b , since a and b both act as catalysts (we require that all reactions are fully catalytic). Because we assume here catalytic flow systems, every molecule is effected by a dilution flow, which is equivalent to the fact that every molecule decays spontaneously. This decay can be simply simulated by choosing randomly a molecule and decreasing its number by one. This can be done by randomly choosing a molecule proportional to its concentration, e.g., the probability to choose a is $19/29$. Destroying a we arrive at $[a] = 19$, $[b] = 5$, $[c] = 5$. How this update is performed, has to be defined in more detail in order to get a precise description of the dynamics. E.g., we may define a reactor size, i.e. a maximum number of molecules in order to prevent unlimited growth.

The set of all possible subsets is called powerset. Instead of writing $A \subseteq M$ we can write $A \in \mathcal{P}(\mathcal{M})$, where $\mathcal{P}(\mathcal{M})$ denotes the set of all possible sets that contain elements from \mathcal{M} .

Example: $\mathcal{P}(\mathcal{M}) = \{\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{c, b\}, \{a, b, c\}\}$. The set $\{\}$, which does not contain any element, is called the empty set. $\mathcal{P}()$ is an operator that take a set as input and returns a set of all possible subsets of that set, including the set itself and the empty set.

Let us notice at that point that a set of molecules represents already a large amount of different states our system that we intend to model can be in. So, taking a set of molecule to describe the current state of a system is a strong abstraction. This level of abstraction does not care, how many copies of molecule a are present, nor does it care about the spatial distribution of those copies, and so on. Despite the high level of abstraction, the number of possible sets is still large. The number of possible sets grows exponentially with the number of possible molecules: $|\mathcal{P}(\mathcal{M})| = 2^{|\mathcal{M}|}$, where $|\mathcal{M}|$ denotes the number of elements in the set \mathcal{M} . For our example (Tab. 1) with $|\mathcal{M}| = 13$ molecules there there are 8096 different sets.

Some sets are more important than others, because we can observe some sets more likely than others. The reason for this is that the system becomes stable more likely when these molecules are present. In order to identify and describe these important sets, we introduce specific conditions, properties, or constraints. These constraints will reduce the set of all possible sets, leaving a set of sets that can be regarded as a representation of the systems organizational structure (Fig. 1, right).

2.2 Closed Sets

The first constraint that we introduce is the (algebraic) closure, which is a classical mathematical concept. Given a set of molecules C that is closed, the closure assures that reactions among molecules in C produce only molecules in C . So, a closed set contains all molecules that can be produced by reactions among those molecules.

Definition 1 (closed set). *A set of molecules $C \subseteq M$ is closed, if for all reactions $(A \Rightarrow B) \in R$, with A is a subset of C , B is also a subset of C .*

Example (closed sets): In our example the closed sets are: $\{\}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, c\}$, $\{c, b\}$, $\{a, b, c\}$. The set $\{a, b\}$ is not closed, since a and b produce c by the reaction $a + b \Rightarrow c$.

Given a set of molecules A , we can always generate its closure. The closure of A is defined as the smallest closed set that contains A . In order to generate the closure of a set A , we simply add one by one all the molecules produced by reactions among elements from A until we cannot add new molecules anymore. A set A is always contained in its closure. With the following definition we have defined an operator $G_{CL} : \mathcal{P}(\mathcal{M}) \rightarrow \mathcal{P}(\mathcal{M})$, which takes a set of molecules as input and returns a closed set of molecules.

Definition 2 (generate closed set). *Given a set of molecules A , we define $G_{CL}(A)$ as the smallest closed set that contains A . We say that A generates the closed set $C = G_{CL}(A)$. We call C the closure of A .*

Example (generate closed set): $G_{CL}(\{a, b\}) = \{a, b, c\}$, because c is produced by reaction $a + b \Rightarrow c$. Further examples are: $G_{CL}(\{a\}) = a$, $G_{CL}(\{\}) = \{\}$. Note that the empty set $\{\}$ is not necessarily always closed. If there is a reaction rule where the left hand side is empty, e.g., $(\Rightarrow c)$ representing an inflow of c , then the empty set is not closed.

The closure implies a union operator and an intersection operator on closed sets. The requirement for a well defined union on closed sets is that the result of the union is closed, too. Imagine that we take two closed sets $\{a\}$ and $\{b\}$ and compute their normal set union $\{a\} \cup \{b\} = \{a, b\}$. Now, $\{a, b\}$ is not closed, because $a + b \Rightarrow c$. So, the normal set union does not fulfil our requirement. Using the generate function, we can easily define a union and an intersection on closed sets. Together with the set of all closed sets, we obtain the lattice of closed sets:

Definition 3 (union and intersection of closed sets). *Given two closed sets A and B , we can define the union and intersection of closed sets by*

$$A \sqcup_{CL} B := G_{CL}(A \cup B), \quad A \sqcap_{CL} B := G_{CL}(A \cap B). \quad (1)$$

The closed set union of two closed sets A and B can be easily calculated by first putting all elements of A and B together and then generating the closed set. Example: Given two closed sets $A = \{a\}$ and $B = \{b\}$, we would like to calculate their closed set union $A \sqcup_{CL} B$. First we compute the normal set union, $A \cup B = \{a, b\}$ and then apply the generate closed set operator G_{CL} to this union, which generates the closed set $\{a, b, c\}$. So, $A \sqcup_{CL} B = G_{CL}(\{a\} \cup \{b\}) = G_{CL}(\{a, b\}) = \{a, b, c\}$.

The set of all closed sets \mathcal{O}_{CL} form a lattice: $\langle \mathcal{O}_{CL}, \sqcup_{CL}, \sqcap_{CL} \rangle$, which is a common algebraic structure (a poset in which any two elements have a greatest lower bound and a least upper bound). The property of closure is important, because the closed set represents the largest possible set that can be reached from a given set of molecules. Furthermore a set that is closed cannot generate new molecules and is in that sense more stable than a set that is not closed.

It is not important here to go into detail what an (algebraic) lattice is. It is sufficient to know the following: A lattice is a mathematical concept. It is defined by a set (here the set of all closed sets), and two operators that map a pair of elements to an element (here the closed set union and intersection, which map two closed sets to a set). When the lattice is not too big, we can draw it nicely as a Hasse diagram (Fig. 1, right). The biggest element is at the top (here the largest closed set, which is always the set containing all possible molecules) and the smallest element is at the bottom (here the smallest closed set). We can also read from the Hasse diagram, how the union and intersection work. A union of two sets leads to the smallest set that is above both sets. An intersection leads to the largest set below the two sets. As can be seen in the Hasse diagram, union and intersection are somehow symmetric. The Hasse diagram implies also a partial order, i.e. a set is smaller than another set, if it is below it.

In summary, given a reaction system by a set of molecules and a set of reaction rules, we can now find all closed sets and display the lattice of closed sets as a Hasse diagram, which provides already information concerning the structure of the reaction system. The set of closed sets is nearly always smaller than the set of all sets. In our example we lost just one set ($\{a, b\}$) by introducing the constraint of closure. Nevertheless, in other systems we usually reduce the number of sets by a much higher rate.

2.3 Self-maintaining Sets

In the same way as we have introduced the closure, we will now introduce the constraint of self-maintenance, which has originated from studies of au-

tocatalytic sets [18]. Later, closure and self-maintenance will define an organization. Note that the definition of self-maintenance given here holds only for catalytic flow systems. For more general reaction systems, which are not considered in this paper, the definition is more complex ([5], Def. 10). Given a set of molecules C that is self-maintaining, the constraint of self-maintenance assures that every molecule of C is produced by at least one reaction among molecules in C . So, a self-maintaining set can produce all molecules inside. It is not necessarily closed.

Definition 4 (self-maintaining set). *Given a catalytic network $(\mathcal{M}, \mathcal{R})$, a set of molecules $C \subseteq \mathcal{M}$ is self-maintaining, if for all molecules $a \in C$, there exists a reaction $(A \Rightarrow B) \in \mathcal{R}$, with $A \subseteq C$ and $a \in B$.*

Example (self-maintaining sets): In our example the self-maintaining sets are: $\{\}$, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{b, c\}$, $\{a, b, c\}$. Note that the set $\{a, b\}$ is self-maintaining but not closed, since a and b produce a , b , and c .

Given a set of molecules A , we can always generate a self-maintaining set that is contained in A . The self-maintaining set generated by A is defined as the biggest self-maintaining set contained in A . In order to generate the self-maintaining set from A , we simply remove one by one all the molecules not produced by reactions among elements from A until we cannot remove molecules anymore. The self-maintaining set generated by A is always contained in A . The self-maintaining set generated by A is be equal to A , iff A is self-maintaining.

Definition 5. (generate self-maintaining set) *Given a catalytic network $(\mathcal{M}, \mathcal{R})$ and a set of molecules $A \subseteq \mathcal{M}$, we define $G_{SM}(A)$ as the biggest self-maintaining set contained in A . We say that A generates the self-maintaining set $C = G_{SM}(A)$.*

With this definition we have defined another operator of the form $G_{SM} : \mathcal{P}(\mathcal{M}) \rightarrow \mathcal{P}(\mathcal{M})$, which takes a set of molecules as input and returns the associated self-maintaining set. Example (generate self-maintaining set): $G_{SM}(\{c\}) = \{\}$, because there is no reaction that can produce c by using only c . Further examples are: $G_{SM}(\{a\}) = a$, $G_{SM}(\{a, c\}) = \{a\}$. Note that the empty set $\{\}$ is always self-maintaining. The operator G_{SM} implies a union operator and an intersection operator on self-maintaining sets in the same way as the closure did.

Definition 6 (union and intersection of self-maintaining sets). *Given a catalytic network $(\mathcal{M}, \mathcal{R})$ and two self-maintaining sets $A, B \subseteq \mathcal{M}$, the self-maintaining set generated by their union and intersection are defined as:*

$$A \sqcup_{SM} B := G_{SM}(A \cup B), \quad A \sqcap_{SM} B := G_{SM}(A \cap B). \quad (2)$$

So, the self-maintaining set intersection of two self-maintaining sets A and B can be easily calculated by first taking only those elements that appear in A

and B together and then generating the self-maintaining set by removing the molecules that are not produced anymore. Example: $A = \{a, c\}$ and $B = \{b, c\}$ are self-maintaining sets of molecules. The intersection is $A \cap B = \{c\}$, which generates the self-maintaining set $\{\}$. So, $A \sqcap_{SM} B = G_{SM}(\{a, c\} \cap \{b, c\}) = G_{SM}(\{c\}) = \{\}$.

Again, the set of all self-maintaining sets \mathcal{O}_{SM} together with the self-maintaining set union \sqcup_{SM} , and the self-maintaining set intersection \sqcap_{SM} form an algebraic lattice. We can now display the set of all self-maintaining sets by a Hasse diagram.

2.4 Chemical Organizations as Closed and Self-maintaining Sets

Now we put both constraints together in order to define the central concept of chemical organization theory: an organization [13]. Note that the term “organization” is a technical term denoting a mathematical object and should not be confused with the term found in economics or the social sciences; in particular there is no relation to Luhmann’s concept of an organization. A chemical organization is more related to the concept of an autopoietic system [6].

Definition 7 (organization, [13]). *An organization is a closed and self-maintaining set of molecules.*

Given an organization $O \subseteq \mathcal{M}$, we know that every molecule of O is produced by at least one reaction among molecules in O and all possible products that can appear by reactions among molecules of O are also contained in O . In our example the organizations are: $\{\}$, $\{a\}$, $\{b\}$, $\{b, c\}$, $\{a, b, c\}$.

Here, given a set of molecules A , we can always generate an organization. We define this organization as the largest organization that can be reached from the set A . In order to generate this organization, we first generate the closure of A , and then generate the self-maintaining set of this closure. So, in order to generate an organization from a set A , we simply add one by one all the molecules produced by reactions among elements from A until we cannot add new molecules anymore. Then we remove one by one all the molecules not produced by reactions among elements from A until we cannot remove molecules anymore. The generated organization is unique, which follows directly from the uniqueness of the closure and generate self-maintaining set operator.

Definition 8 (generate organization). *Given a set of molecules A , we define $G(A)$ as the biggest organization that can be reached from A . Formally, $G(A) := G_{SM}(G_{CL}(A))$. We say that A generates the organization $O = G(A)$.*

Example (generate organization): $G(\{a, b\}) = \{a, b, c\}$, because there is no reaction that can produce b by using only b . Further examples are: $G(\{c\}) = \{\}$, $G(\{a\}) = \{a\}$, $G(\{a, c\}) = \{a\}$. As before, the operator G implies a union

operator and an intersection operator on organizations in the same way as the closure did.

Definition 9 (union and intersection of organizations). *Given two organizations A and B . The organization union and intersection is defined as the organization generated by their set union and intersection, respectively:*

$$A \sqcup B := G(A \cup B), \quad A \sqcap B := G(A \cap B). \quad (3)$$

Example: $A = \{a\}$ and $B = \{b\}$ are organizations. $A \sqcup B = \{a, b, c\}$. $A \sqcap B = \{\}$.

The set of all organizations \mathcal{O} (of a catalytic network) together with the organization union \sqcup , and the organization intersection \sqcap form an algebraic lattice $(\mathcal{O}, \sqcup, \sqcap)$.

We can now display the set of all organization by a Hasse diagram, which we may call the organizational structure of the reaction system (Fig. 1, right). An organization represents an important combination of molecular species, namely those that are likely to be observed in a reaction vessel on the long run. A set of molecules that is not closed or not self-maintaining would not exist for long, because new molecules can appear or some molecules would vanish, respectively.

2.5 Dynamics

So far we have analyzed the reaction system statically. This means that time has not played any role in our analysis. However, the result of the static analysis has strong implications for the potential dynamics of the reaction system. One such implication is expressed by a theorem that relates fixed points (stationary states) to organizations, and by doing so, underlines the relevancy of organizations. The theorem says that, given the dynamics of the reaction system by a continuous ordinary differential equation (ODE) of a form that is commonly used to describe the dynamics of reaction systems and that obeys the rules given by the reaction system, then every fixed point of this ODE is an instance of an organization. This implies that we can only have a stationary state with exactly those molecules that form an organization. It is not possible to find a stationary state with a combination of molecules that are not an organization. And this implies further that only organizations are candidates for autopoietic systems. However, an organization is not necessarily an autopoietic system, since a stationary state is not necessarily stable. In an instable stationary state the system can reside for ever, as in a stable stationary state, but a tiny perturbation would cause the system to move away from this state towards a different (stable) attractor.

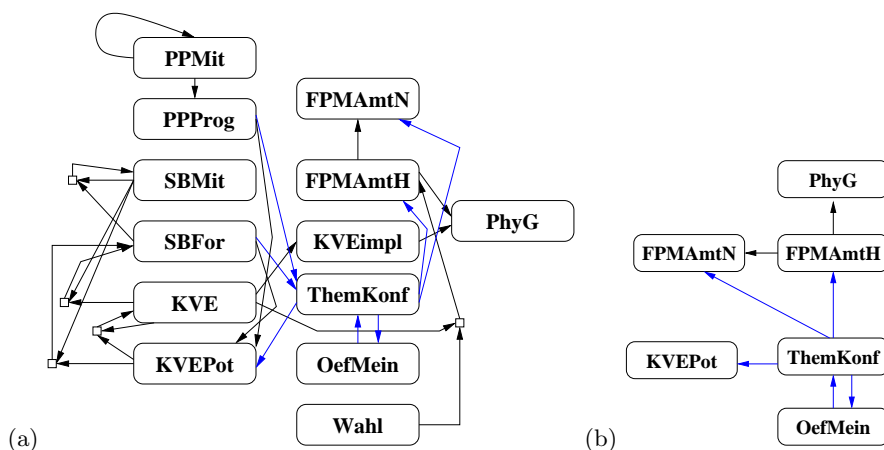


Fig. 2. (a) The toy model of the political system inspired by Luhmann. (b) The smallest non-empty (chemical) organization. An arrow represents a so called catalytic reaction, that is a reaction where the educts are not consumed when producing the products. Or in other words, a decision is not “used-up” when it produces another decision. For example, if there is one PPMit (political party member) there can be two of them in the next time step according to the rule $\text{PPMit} \Rightarrow \text{PPMit}$.

3 A Toy Model of the Political System

In this section we sketch a reaction network model for the political system inspired by Luhmann [14]. This model should help to illustrate how chemical organization theory can be applied to such systems. Note that this section is preliminary and should only illustrate how the theory might be applied. The model used is not validated and thus our study does not allow to draw conclusions concerning the real political system. However, the toy model we generate demonstrates that our theory can uncover interesting “hidden” structures that are not obvious when looking at the network.

3.1 The Chemical Model

The political system is modeled as an catalytic network, which consists of a list of molecules and a list of reaction rules (i.e. production rules). A molecule represents a specific communication, which in the political system is a decision according to Luhmann [14]. There are 13 different decisions (Table 1) and 20 reaction rules (Table 3). Figure 2 (a) shows a graphical representation of the model.

Figure 2 (a) illustrates that it not easy to identify structures within the model, whose graphical representation is already optimized to some extent. In order to elucidate the structure hidden in this network, we will apply chemical organization theory in the next section.

Symbol	Meaning
	POLITICAL PARTIES
PPMit	Political party members (Germ.: politische Partei, Mitglied): Political parties decide decision prerequisites. PPMit represents the decision that leads to new party members.
PPProg	Political party program (Germ.: politische Partei, Programm): Decision concerning the program of the political party.
	SOCIAL MOVEMENT (e.g. labor union)
SBMit	Social movement members (Germ.: soziale Bewegung, Mitglieder)
SBFor	Political demands of a social movement (Germ.: soziale Bewegung, Forderung)
	CONSENSUS DECISIONS
KVE	Collectively binding decisions (Germ: kollektiv verbindliche Entscheidungen): E.g. laws, regulations, state of emergency.
KVEpot	Potential collectively binding decisions (Germ.: potenzielle kollektiv verbindliche Entscheidungen): E.g., draft of a law or draft of a regulation.
	DECISION CONCERNING OFFICES AND POSITIONS
FPMamtH	Formal political power (high) (Germ.: formale politische Macht, hohe Ämpter): E.g. chancellor, president, major, member of parliament.
FPMamtN	Formal political power (low) (Germ.: formale politische Macht, niedrige Ämpter): E.g. positions in administration departments.
	OTHER
KVEimpl	Implementation of collectively binding decisions (Germ.: kollektiv verbindliche Entscheidungen, Implementierung): KVEimpl represents the concrete implementations made by administration departments.
PhyG	Physical force (Germ.: physische Gewalt): Decisions concerning the use of physical force, such as military actions.
ThemKonf	Thematic conflict (Germ.: thematischer Konflikt). The decision to chose a specific conflict as a subject, e.g. for discussion.
OefMein	Public Opinion (Germ.: öffentliche Meinung).
Wahl	Public elections (Germ.: Wahl).

Table 1. The set of molecules constituting the toy model of the political system. A molecule represents a communication, or more particular, a decision [14].

3.2 Preliminary Results

Applying chemical organization theory to our model, 10 organizations are found. Figure 3 shows the lattice of organizations. The smallest organization (Fig. 3, bottom) contains no molecules (decision), because nothing is spontaneously created. The largest organization contains all decisions but **Wahl** (public elections). As we can see in Fig. 2, **Wahl** is not created by any rule, so it cannot be part of any organization, because an organization must be self-maintaining (i.e. all its elements must be produced). We can take the observation that **Wahl** does not appear in any organization to check and improve the model.

The smallest non-empty organization is

Reaction rule	Meaning
	POLITICAL PARTIES
$PPMit \Rightarrow PPMit$	Members of political parties recruit further members. Spontaneous formation of political parties is not considered here.
$PPMit \Rightarrow PPProg$ $PPPprog \Rightarrow KVEpot$	Members of a political party decide the party program. A party program implies potential collectively binding decisions.
	SOCIAL MOVEMENT
$SBMit + SBFor \Rightarrow SBMit$	New members of a social movement are recruited by present members of the social movement $SBMit$ together with their demands $SBFor$. This rule means that both, members $SBMit$ and their demands $SBFor$ are necessary to get more members.
$SBMit + KVE \Rightarrow SBFor$	Demands of social movements (e.g. lowering the tax) can be stimulated by collectively binding decisions (e.g. increase tax)
$SBMit + KVEpot \Rightarrow SBFor$	Demands of social movements $SBFor$ (e.g. not to increase the tax) can be stimulated by potential collectively binding decisions (e.g. the plan to increase tax)
	POLITICAL POWER
$FPMamtH \Rightarrow FPMamtN$	High political offices decide for positions in lower administration.
$SBFor \Rightarrow KVEpot$	The demands of social movements $SBFor$ can also generate potential collectively binding decisions $KVEpot$.
	POLITICAL PROCEDURES
$Wahl + KVE \Rightarrow FPMamtH$	Democratic election $Wahl$ leads to decisions concerning high offices like chancellor or president. Elections require laws KVE
$KVEpot + KVE \Rightarrow KVE$	This rule models law-making. In order to pass a law, laws concerning the law-making process are required. Furthermore potential drafts $KVEpot$ are needed.
$KVE \Rightarrow KVEimpl$	Laws are implemented by administration departments.
	PHYSICAL FORCE
$KVEimpl \Rightarrow PhyG$	This rule captures the physical force executed by the police. Note that it depends on the particular political systems how the usage physical force is decided.
$FPMamtH \Rightarrow PhyG$	Decision for using physical force by high offices $FPMamtH$. E.g. military actions.

Table 2. The set of reaction rules constituting the toy model of the political system.

Reaction rule	Meaning
	POLITICAL THEMATIC CONFLICT
$\text{SBFor} \Rightarrow \text{ThemKonf}$	A thematic conflict can be created by demands of social movements, such as labor unions.
$\text{PPPprog} \Rightarrow \text{ThemKonf}$	The program of political parties lead to a thematic conflict.
$\text{OefMein} \Rightarrow \text{ThemKonf}$	A thematic conflict can also be created by a public conflict (“autistic world of the political system”).
$\text{ThemKonf} \Rightarrow \text{KVEpot}$	A thematic conflict generates further potential collectively binding decisions.
$\text{ThemKonf} \Rightarrow \text{FPMamtH}$	A thematic conflict can, e.g., force a politician to resign.
$\text{ThemKonf} \Rightarrow \text{FPMamtN}$	A thematic conflict ThemKonf can also influence decisions concerning positions in administration departments FPMamtN .
$\text{ThemKonf} \Rightarrow \text{OefMein}$	Thematic conflict ThemKonf stimulations public opinion OefMein .

Table 3. The set of reaction rules constituting the toy model of the political system.

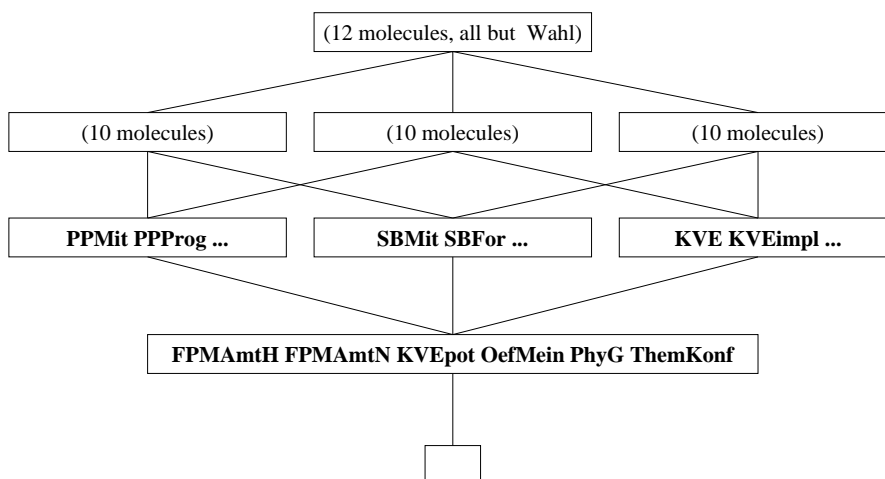


Fig. 3. The lattice of organization of the toy model of the political system. The smallest organization is empty and drawn at the bottom. The number of elements is growing when going from the bottom to the top organization, which contains 12 elements (i.e. molecules, decisions, communications). Only molecules that are new are depicted (cf. Fig. 1, right).

$$O_1 = \{\text{FPMamtH}, \text{FPMamtN}, \text{KVEpot}, \text{OefMein}, \text{PhyGThemKonf}\}. \quad (4)$$

It contains a self-maintaining sub-set $\{\text{OefMein}, \text{ThemKonf}\}$, which generates (Def. 2.4) the whole organization. Note that the sets $\{\text{OefMein}\}$ and $\{\text{ThemKonf}\}$ are the smallest generators of that organization. It is interesting to note that KVE (collectively binding decisions like laws) are not part of this

organization. Is this intended by the modeler? If not, the result can again be taken to improve the model.

Having identified organizations, we can continue with our analysis by asking, which kinds of decisions are required for a particular organization to exist, or which kind of decisions can we remove so that the remaining set still generates the original organization. Using the terminology of our theory, we can look for the internal generators. For example, a decision that has only inputs is not required. Removing all nodes that have only incoming edges we would arrive at a smaller internal generator. In Fig. 2 we can see that removing the decision `PhyG` (physical force) would result in a set that still generates the original organization. As opposed to this, members of a political party `PPMit` are necessary, e.g., for the largest organization. If we remove `PPMit` from an organization containing them, the remaining set of decisions generates a smaller organization without `PPMit` and without `PPPProg`.

4 Discussion

The results presented here are preliminary. Although we have derived our toy model based on Luhmann's theory of the political system, the derived reaction network is unrealistic. However, the aim of this paper is to illustrate the method and to demonstrate how a higher level of precision in formulating social theories can be obtained and tools like chemical organization theory can help to handle such formalisms.

In the context of our theory, there are a couple of interesting open questions: What kind of decisions are required for a particular organization to be generated? How do the generators of an organization look like? How do the smallest internal generators of an organization look like? This may characterize the potential stability of the system.

Furthermore we may map real data to the set of organizations. Assume that we have data on the time evolution of the size of the different elements of the political system (e.g. financial strength of social movements) over time. We can now measure whether and with what intensity a (chemical) organization is present in a certain state in time. This provides a new way to project a high dimensional system to a low dimensional system (i.e., the lattice of organizations), which may give further insights into the structure of the network.

Acknowledgment

We acknowledge support by Federal Ministry of Education and Research (BMBF) Grant 0312704A and by the German Research Foundation (DFG) Grant Di 852/4-1.

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