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# A simple model of interactions between Belousov-Zhabotinsky droplets

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## **Abstract**

Lipid covered droplets containing solution of reagents of oscillatory Belousov-Zhabotinsky reaction and surrounded by hydrocarbons form an interesting system for studying information processing operations that can be directly executed by a chemical medium. Activa-

tions are transmitted between droplets in contact so the ensemble of droplets can be regarded as a prototype of a neural network. In the paper we introduce a simple mathematical model that can be used to simulate the time evolution of interacting droplet system in realistic experimental conditions. The chemical dynamics is described by the recently introduced, two variable model that correctly predicts the period of oscillations for a large class of conditions at which droplets are investigated. The parameters describing diffusion of activator and its transport through separating lipids layers are fitted to match the experimental results. We apply the model to determine the stable modes of coupled oscillations in two identical droplets.

## 1 Introduction

Unconventional computing is a field of interdisciplinary research focused on interpreting the natural time evolution of a physical system as a sequence of information processing operations [1, 2]. A special attention within the unconventional computing is given to chemical systems because chemical processes are responsible for sensing, communication and decision making in living organisms. Studies on unconventional computation with chemical media are concerned with processes at various spatial and temporal scales. The systems considered for information coding and processing include stationary states of reaction networks [3, 4], macroscopic spatio-temporal structures appearing in reaction-diffusion processes [5] as well as selective bonding phenomena at the level of individual molecules [6, 7].

It has been demonstrated that relatively simple chemical systems can

imitate the behavior of biological neural networks [3, 8]. A typical nerve system is build of non-linear elements (neurons) that are linked together via communication channels in which signals propagate and form a specific network. A chemical medium showing similar behavior can be generated with a clever geometrical distribution of excitable and nonexcitable regions. Studies on a chemical equivalent of neuron networks, for example based on photosensitive Belousov-Zhabotinsky (BZ) medium with excitability controlled by illumination level [8], have shown that the geometrical structure of fragments characterized by different excitabilities plays equally important role as the chemical kinetics. Even for quite simple excitable kinetics we can construct logic gates and build a universal computer by setting the proper structure of illuminated and non-illuminated regions [10]. However, in all applications of photosensitive medium to information processing a human factor played the crucial role because an experimentalist manually fixed the geometrical distribution of illumination levels or operated the devices used to control the experiment.

It seems much more challenging to find a chemical medium in which the structure required for information processing is self-generated at properly selected nonequilibrium conditions. A medium composed of droplets containing the water solution of reagents of BZ reaction that are separated by the oil-phase is one of the systems in which self-realization of information processing functions seems possible [11, 12]. Our experiments were concerned with droplets covered by a layer of phospholipids and surrounded by hydrocarbons [13]. The diameter of droplets in such medium is by an order of magnitude larger than the those studied by Epstein group [11, 12, 14, 15, 16]. The pres-

ence of lipid monolayer stabilized droplets mechanically so the structures they form remained unchanged for hours. The time evolution of excitations in such medium, that can be interpreted as a sequence of information processing operations, is determined by the kinetics of Belousov-Zhabotinsky reaction. In the case of ferroin catalyzed reaction two states of the medium, corresponding to large concentrations of the catalyst in the reduced or the oxidized forms can be easily distinguished by the color. In the following we regard the state corresponding to a high concentration of the oxidized catalyst (blue color of solution) as the excited one. In a droplet containing oscillatory solution of BZ-reagents the excited state periodically re-appears. We have observed ( see Fig. 1) that excitations of one droplet can influence its neighbors via exchange of reagents through separating lipid layers. Therefore, the structure of multiple droplets can be regarded as a prototype of a neural network, where individual droplets play the role of nonlinear elements (neurons) that are linked together and communicate. Fig. 1 shows that in a typical experiment droplet excitations are not homogeneous. For droplets with diameters exceeding  $1mm$  we frequently observed pulses of excitation propagating inside them. In such case the excited region covered a small fraction of droplet volume.

Experiments with multiple droplets are difficult. The theoretical studies and in-silico simulations of droplet structures with potential interest for information processing operations are helpful because they allow to test a large number of configurations and concentrations of reagents and select the most interesting for real experiments. In order to translate theoretical predictions from the virtual world into reality we need a mathematical model that can be

related quantitatively with experimental conditions. The aim of this paper is to propose a simple yet realistic model that can be used to simulate the time evolution of multiple droplet system observed in typical experimental conditions with Belousov-Zhabotinsky reaction. The model we present describes both spatiotemporal dynamics inside a single droplet and interactions between droplets and it is as simple as the Oregonator [17]. The droplets in our experiments are relatively large and the geometry of excitations propagating inside a droplet is important for information processing functions executed by the medium [18, 19]. The simulations of droplets have to base on partial differential equations. A simple model of chemical kinetics can significantly reduce the time of calculations if compared with multivariable ones.

The paper is organized as follows. In the subsequent section we refine the values of parameters describing chemical kinetics to obtain a good agreement with periods of oscillations for concentrations of reagents used in experiments. We estimate the diffusion of activator using the observed excitation velocity. Activator degradation rate in the lipid bilayer is fixed to describe the transformation of excitation frequency in communication between droplets. In the next section we use the model to estimate the stability of oscillations in two coupled identical droplets. We show that both in-phase and anti-phase oscillations are stable, and we estimate the basin of attraction for each mode.

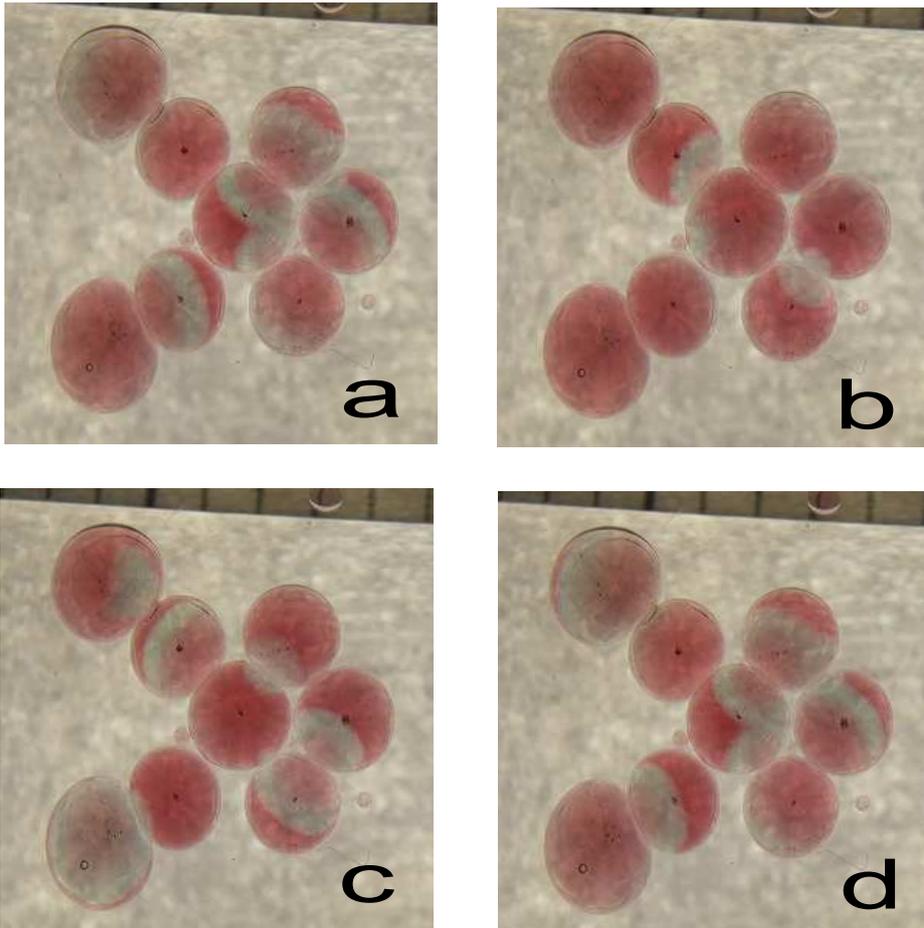


Figure 1: The time evolution of excitations (light areas) in a system of interacting droplets. The diameter of the largest droplets is 2 mm. The time interval between consecutive frames is 6 sec.

## 2 Spatio-temporal dynamics of BZ-droplets

In this section we discuss two types of spatiotemporal phenomena observed in BZ-droplets: pulse propagation within a single droplet and transformation of frequency when train of excitations is transmitted from one droplet to another. Their analysis allows us to extract the values of model parameters. We discuss the procedures of parameter fitting. The diffusion of activator  $D_x$  in Eq.(6) is selected to give a correct value of pulse velocity observed in experiments. The model for activator degeneration inside the double layer is constructed to explain transformation of frequency of excitations passing between droplets containing solutions of reagents that produce oscillations with different periods.

### 2.1 Simple models of reaction kinetics.

We describe the chemical kinetics inside a droplet using Rovinsky-Zhabotinsky-like models [21, 22]. Such models are useful for in-silico experiments because they include a small number of differential equations, but still the equation parameters can be fitted such that the results are in a good agreement with experiment. The three-variable model [23] is based on the following kinetic equations:

$$\frac{\partial x}{\partial t} = \epsilon_1 h_0 N x - \epsilon_2 h_0 x^2 - 2 \frac{\alpha \gamma \epsilon_1}{\beta} h_0 x y + 2 \frac{\alpha \gamma \epsilon_1 \mu}{\beta} h_0 N y \quad (1)$$

$$\frac{\partial y}{\partial t} = q \beta \frac{M * K}{h_0} \frac{z}{1 - z} - \gamma h_0 x y - \gamma \mu h_0 N y + M * K \quad (2)$$

$$\frac{\partial z}{\partial t} = \frac{h_0 N}{C} x - \alpha \frac{K * M}{C h_0} \frac{z}{(1-z)} \quad (3)$$

where the model variables  $x$ ,  $y$  and  $z$  denote scaled concentrations of  $HBrO_2, Br^-$  and  $Fe(phen)_3^{3+}$  respectively. The symbols  $C$ ,  $K$ ,  $M$  and  $N$  denote the total concentration of the catalyst ( $C = [Fe(phen)_3^{2+}] + [Fe(phen)_3^{3+}]$ ) and concentrations of  $KBr$ ,  $CH_2(COOH)_2$  and  $NaBrO_3$  respectively. Such choice of model parameters reflects the fact that in our experiments BZ-droplets were prepared with water solution of sulfuric acid, sodium bromate, potassium bromide, malonic acid and ferroin [13, 23] and the concentrations of these reagents are precisely known when an experiment starts.  $h_0$  is the Hammett acidity function of the solution. For the concentrations of sulfuric acid used in our experiments it can be approximated as 1.3 times the concentration of  $H_2SO_4$ . The other parameters of the model:  $\alpha, \beta, \gamma, \epsilon_1, \epsilon_2, \mu$  and  $q$  do not depend on concentrations of reagents used to prepare BZ-droplets. Their values are fitted to obtain the best match with periods of oscillations measured in experiments.

The three variable model (Eqs.(1-3)) can be simplified if we assume that the relaxation of  $y$  is fast and the stationary value of  $y$  is immediately approached. Calculating the steady value of  $y$  from Eq.(2) and substituting it to Eq.(1) we obtain the two variable model:

$$\frac{\partial x}{\partial t} = \epsilon_1 h_0 N x - \epsilon_2 h_0 x^2 - 2\alpha \epsilon_1 M * K \left( \frac{1}{\beta} + q \frac{1}{h_0} \frac{z}{1-z} \right) \frac{x - \mu N}{x + \mu N} \quad (4)$$

$$\frac{\partial z}{\partial t} = \frac{h_0 N}{C} x - \alpha \frac{K * M}{C h_0} \frac{z}{(1-z)} \quad (5)$$

with parameters  $\alpha, \beta, \epsilon_1, \epsilon_2, \mu$  and  $q$ .

Both models presented above can be used to predict the time evolution of their variables in the BZ-droplets for typical concentrations of reagents used in experiments. As it can be seen on Fig. 1 oscillations inside droplets are not spatially homogeneous and excitations can be transmitted from one droplet to another when droplets are in contact. For completeness of mathematical model we have to describe both phenomena. Having a limited amount of experimental results we decided to make a model that is as simple as possible with a small number of adjustable parameters. The oxidized form of catalyst ( $z$ ) is a large ion (it contains three bulky phenanthroline ligands ) and we neglect its mobility if compared to the other reagents. Introducing diffusion within the three variable model requires self-diffusion constants for both  $x$  and  $y$  as well as cross-diffusion parameters. At the moment we have not collected enough experimental data to give precise values of diffusions, so we focus our attention on the two variable model with a single self-diffusion constant  $D_x$  for  $x$ . Within these approximations equations describing spatio-temporal effects inside a droplet are:

$$\frac{\partial x}{\partial t} = \epsilon_1 h_0 N x - \epsilon_2 h_0 x^2 - 2\alpha \epsilon_1 M * K \left( \frac{1}{\beta} + q \frac{1}{h_0} \frac{z}{1-z} \right) \frac{x - \mu N}{x + \mu N} + D_x \Delta x \quad (6)$$

$$\frac{\partial z}{\partial t} = \frac{h_0 N}{C} x - \alpha \frac{K * M}{C h_0} \frac{z}{(1-z)} \quad (7)$$

The ion representing the oxidized form of catalyst seems to be too large to cross lipid bilayer separating droplets. Therefore in the model we use Eq.(7) with no-flow conditions at droplet boundary. In order to get communica-

tion between droplets in the model based on Eqs.(6-7) we have to assume that  $HBrO_2$  can migrate through the double lipid layer. We introduce a mechanism of activator transport between droplets and fit its parameters.

## 2.2 The parameters of two variable model

The parameters of the two variable model (Eqs.(4-5)) given in [23] were obtained after optimization of periods over a large number of experiments performed with different BZ-mixtures. In experiments discussed below the concentration of sulfuric acid was around  $0.5M$ . Selected results for the period of homogeneous oscillations within a droplet as a function of acidity of the medium are illustrated in Fig. 2. The experimental results are marked by crosses. The three variable model with parameters from [23] gives quite good estimation of the observed period ( empty circles), but, as we have mentioned, its application for description of spatiotemporal effects requires a model of diffusion for both  $HBrO_2$  and  $Br^-$ . The periods predicted by the two variable model with parameters from [23] ( $\alpha = 2.6 * 10^{-4}$ ,  $\beta = 200.$ ,  $\epsilon_1 = 4000.$ ,  $\epsilon_2 = 5800.$ ,  $\mu = 2.1 * 10^{-5}$  and  $q = 0.88$ ) are marked with empty squares. For these parameters the calculated period is approximately twice as long as that observed in experiments. If a kinetic model fails to predict the period then one should not expect to estimate correctly the value of activator diffusion coefficient by comparing calculated velocity with this measured in an experiment. In order to improve the two variable model we re-fitted its parameters by optimizing calculated periods for concentrations of sulfuric acid used in experimental studies on spatiotemporal effects. The optimization

procedure was the same as described in [23] and over 20 different sets of concentrations of BZ-reagents were considered. The re-adjusted values of parameters for two variable model are:  $\alpha = 2.6 * 10^{-4}$ ,  $\beta = 1000.$ ,  $\epsilon_1 = 1200.$ ,  $\epsilon_2 = 6700.$ ,  $\mu = 1.6 * 10^{-5}$  and  $q = 0.51$ . As it can be seen, they are not much different from the original values, but they lead to much better agreement with experiment (the solid line on Fig 2). We will use these values in the subsequent calculations.

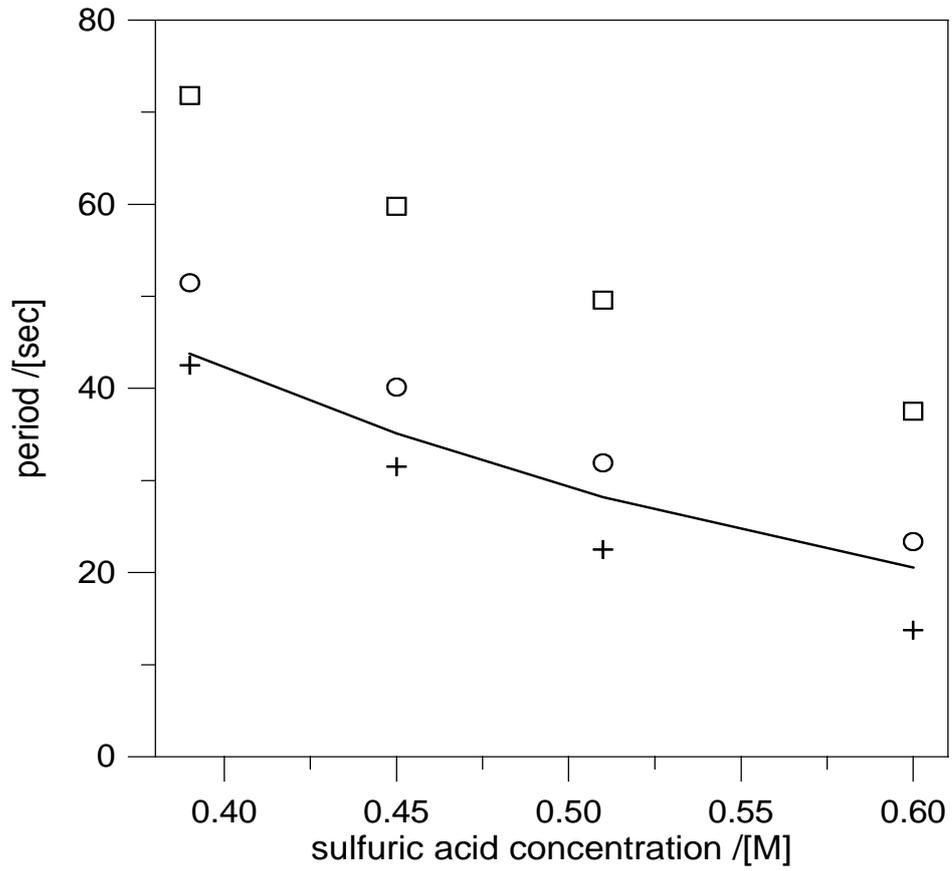


Figure 2: Period of oscillations as a function of concentration of sulfuric acid for  $[NaBrO_3] = 0.45M$ ,  $[CH_2(COOH)_2] = 0.35M$ ,  $[KBr] = 0.06M$  and  $C = 0.0017M$ . Crosses show the experimental results, empty circles periods calculated using the 3-variable model with parameters given in [23], empty squares periods calculated with the 2-variable model with parameters given in [23]. The solid line plots results of the 2-variable model with the re-fitted parameters.

### 2.3 The estimation of diffusion coefficient

In experiments we measure velocity of excitation pulses propagating in elongated droplets. The solution of BZ-reagents is denser than the surrounding oil phase so droplets placed in a narrow trench at the bottom of a cuvette assume elongated shapes. If a droplet is long enough then an excitation pulse generated at one of droplet ends reaches the stable form and velocity within the droplet. The observed value of velocity can be compared with the numerical solution of Eqs.(6-7)) and the value of  $D_x$  for which the numbers match can be extracted. We studied wave propagation in droplets that were 6 mm long and 2 mm wide (see Fig. 3 a). The excitation pulses propagated along the droplet for more than 20 sec with a constant velocity (see Fig. 3 b). To obtain unidirectional propagation of pulses we introduced a pacemaker ( a piece of stainless steel seen as a black triangle at the left end of droplet in Fig. 3(a)). The stainless steel acted similarly to a silver wire and reduced the concentration of  $Br^-$  ions that inhibit the reaction. The pacemaker becomes a source of pulses with high concentration of the oxidized catalyst propagating to the left. Two excitation pulses can be seen in Fig. 3(a), one at the right end of the droplet and another close to the large gas bubble. As the result of forced activation of the medium the excitation maxima re-appear at a given point more frequently than in a homogeneous medium without the pacemaker. The observed average time between maxima of oxidized catalyst in the elongated droplet was 21sec. A small (diameter 1.3mm) separated droplet seen in the left bottom corner of Fig. 3(a). It contained the same solution of BZ-reagents and the measured period of oscillations

inside it was about 31 sec. For  $[H_2SO_4] = 0.45M$ ,  $[NaBrO_3] = 0.45M$ ,  $[CH_2(COOH)_2] = 0.35M$ ,  $[KBr] = 0.06M$  and  $C = 0.0017M$  the measured value of excitation velocity is 0.2mm/sec.

In order to find the velocity of a propagating pulse and fix the value of the diffusion coefficient  $D_x$  in Eqs.(6), such that the calculated velocity is close to the observed value we simulated propagation of excitations in one dimensional medium. A grid composed of 6000 points was used. Calculations were performed using the direct Runge-Kutta method with  $dt = 10^{-6}$  and  $dx=0.001$ . For concentrations of reagents used in the experiment the period of oscillations predicted by the model based on Eqs.(4-5)) with re-fitted parameters is equal to 33.9sec. This value agrees well with the period of oscillation observed in the separated small droplet, but it is much longer than the period of forced oscillations (21 sec) that generate excitation pulses propagating in the elongated droplet (cf. Fig. 3(a)). The velocity of a train of pulses depends on the frequency, so in simulations we considered a source of excitation pulses with similar frequency as observed in the experiment. We assumed that concentration of  $KBr$  at 1000 leftmost points of the grid is reduced to 0.03M and the concentration of  $KBr$  at the remaining points equals to 0.06M as in the experiment. For the reduced concentration of  $KBr$  the calculated period of oscillations is 25.7 sec which is close to the period between subsequent pulses observed in the experiment. The calculated time evolution of  $z$  is illustrated in Fig 4. As expected the rapid oscillations in the left part of the grid act as a source of propagating excitation pulses. After a few initial oscillations the evolution of the whole system becomes dominated by pulses propagating from the left. For  $D_x = 3.4 * 10^{-7} cm^2/sec$  the

pulse velocity is 0.2 mm/sec. This value of diffusion coefficient is used in the subsequent calculations. In many papers on simulations of BZ-medium the activator diffusion coefficient is approximated by  $10^{-6}cm^2/sec$  and this value is not much different from the result of our analysis.

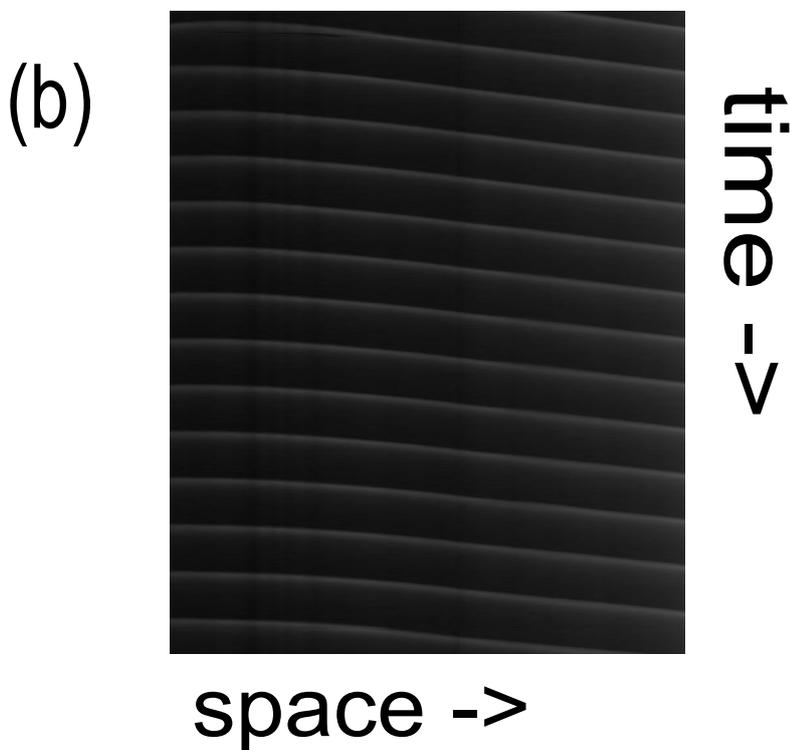
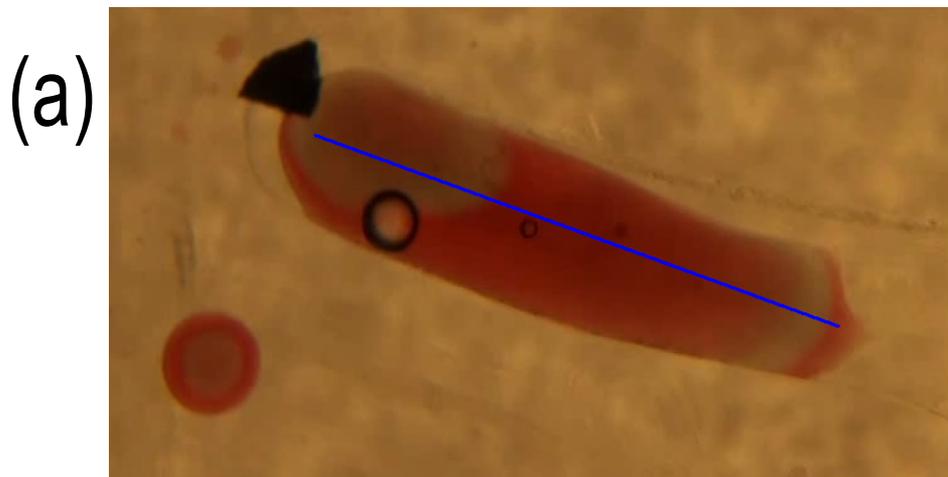


Figure 3: Fig. 3(a) illustrates a configuration of droplets in a typical experiment on pulse propagation. The initial concentrations of reagents were  $[H_2SO_4] = 0.45M$ ,  $[NaBrO_3] = 0.45M$ ,  $[CH_2(COOH)_2] = 0.35M$ ,  $[KBr] = 0.06M$  and  $C = 0.0017M$ . There are two droplets on the snapshot. The diameter of the small droplet is  $1.3mm$ . The upper elongated droplet is  $6mm$  long and  $2mm$  wide. In this droplet pulses of excitation (corresponding to the oxidized catalyst) propagate along the droplet from left to right. At the snapshot two such pulses can be noticed; one above the gas bubble and one close to the right end of the droplet. Fig. 3(b) shows the green component of the space-time plot along the line marked blue on Fig. 3(a). The horizontal space size is  $5.6mm$  and the vertical time scale corresponds to  $300sec$ .

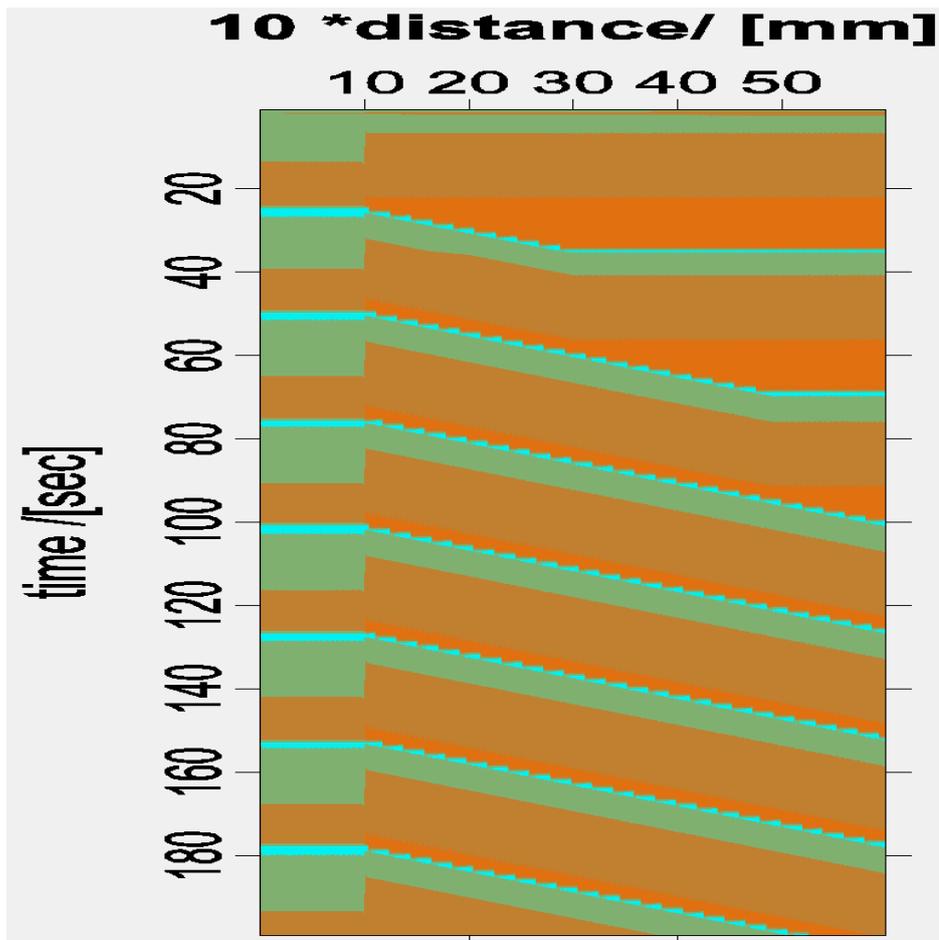


Figure 4: The time evolution of propagating excitations in one-dimensional medium described by Eqs.(6-7)). Blue color marks high concentration of the oxidized catalyst. A low concentration of  $KBr$  is assumed in the left part, so excitation pulses originating from this region dominate in the medium.

## 2.4 A simple model for communication between droplets

Fig. 1 illustrates that droplets in contact communicate: excitations can cross the lipid layer and enter neighboring droplets. The excitatory type of coupling between droplets was frequently observed in our experiments. In a system of multiple droplets the region with the fastest oscillations becomes a pacemaker that dominates the spatio-temporal evolution. We performed a number of experiments in which concentrations of reagents in interacting droplets were different. In many cases we observed transformation of frequency when excitations were transmitted between droplets. Frequency transformation is common when two nonlinear systems communicate [24, 25]. For example, it was observed in an array of small BZ-droplets separated by an oil phase and communicating with an inhibitory messenger [15]. One of our experiments showing frequency transformation with excitatory coupling is illustrated in Fig. 5. Concentrations in the topmost droplet were  $[H_2SO_4] = 0.6M$ ,  $[NaBrO_3] = 0.225M$ ,  $[MA] = 0.26M$ ,  $[KBr] = 0.06M$  and the catalyst  $0.0017M$ . For such concentrations the period of oscillations in a separated droplet is about  $120sec$ . Two other droplets contained the solution of BZ-reagents where  $[H_2SO_4] = 0.6M$ ,  $[NaBrO_3] = 0.45M$ ,  $[MA] = 0.35M$ ,  $[KBr] = 0.06M$  and the catalyst  $0.0017M$ . Here oscillation period was about  $20sec$ . The space-time diagram in Fig. 5 illustrates position of excitation pulses along the vertical line going centrally through the droplets. The dark, wedge-shaped areas mark gas bubbles seen on the snapshots above. There was no pacemaker in this experiment. After the initial period the evolution of the upper droplet was governed by excitation pulses coming from

rapidly oscillating droplets below. During the experiment we observed the frequency transformation: only a part of excitations that appeared in the lower droplets activate the upper one. The space time plot shows transition from 3:1 to 2:1 frequency transformation, which is probably related to the decrease in frequency of arriving excitation pulses. Although the time evolution of concentrations in lower droplets was not spatially uniform, the excitations arriving to the bilayer separating the upper droplet were similar to that generated by a homogeneous evolution. In both cases the wave vector of arriving excitations was perpendicular to the boundary.

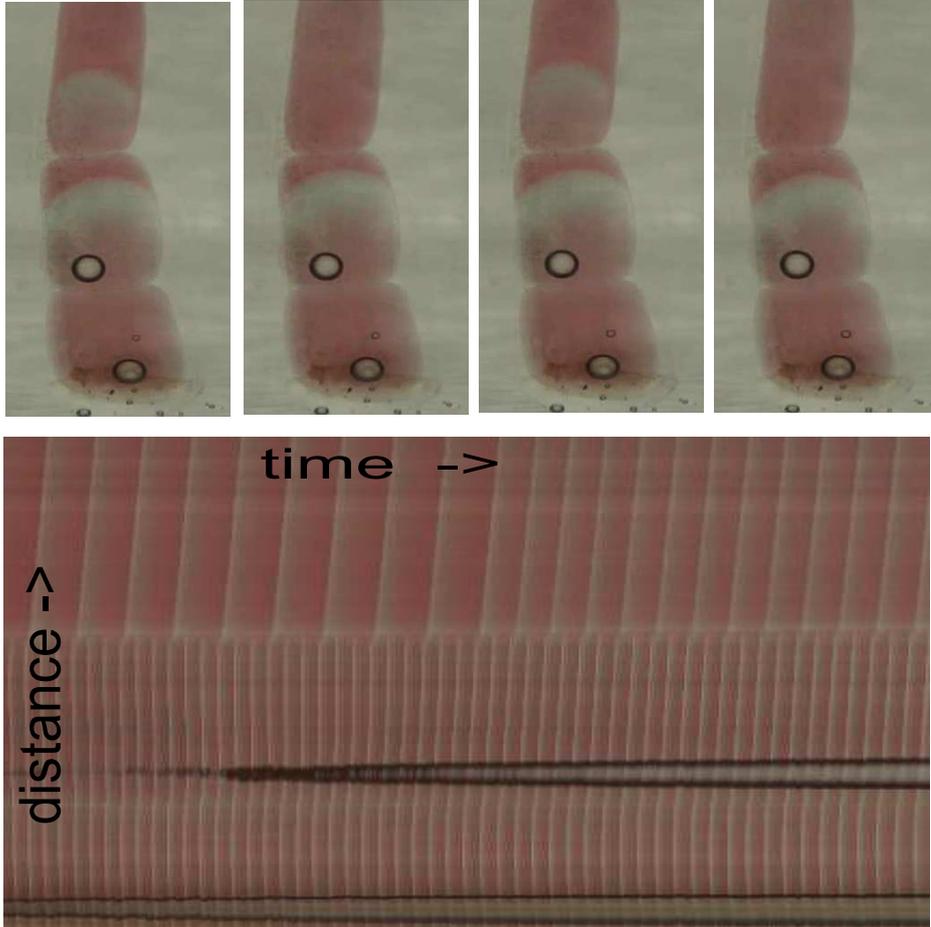


Figure 5: The time evolution of excitations (light areas) in three linked droplets. The width of droplets is  $1.5\text{mm}$ . Concentrations in the top-most droplet were  $[H_2SO_4] = 0.6M$ ,  $[NaBrO_3] = 0.225M$ ,  $[MA] = 0.26M$ ,  $[KBr] = 0.06M$ ,  $catalyst = 0.0017M$ ; in two droplets below  $[H_2SO_4] = 0.6M$ ,  $[NaBrO_3] = 0.45M$ ,  $[MA] = 0.35M$ ,  $[KBr] = 0.06M$ ,  $catalyst = 0.0017M$ . The upper row of snapshots illustrates 2:1 frequency transformation observed for long times. The space-time diagram below shows transition from 3:1 to 2:1 frequency transformation observed in the system. A growing gas bubble inside the mid droplet is represented by a wedge-like region expanding to the right.

The observed frequency transformation should be predicted by a correct model describing interactions between droplets. The inhibitor  $z$  in the model based on Eqs.(6-7)) represents the concentration of a large ionic complex. We can assume that the complex is too bulky to penetrate a lipid bilayer. Therefore, within the two variable model, only migration of activator between droplets can be responsible for transmission of excitations. Let us consider two droplets that are in contact. The double layer should be penetrable for the activator, because otherwise no communication occurs. We described the double layer as a medium separating the droplets (see Fig. 6). Migration of  $HBrO_2$  between droplets and the double layer is represented by a diffusion process with the diffusion coefficient  $D_l$ . We have no experimental data that allow us to estimate  $D_l$ . In order to introduce as few new parameters in the model as possible we assume that the diffusion coefficient  $D_l$  is the same as the diffusion coefficient within a droplet and so equal to  $D_x$  estimated in the previous section.

The numerical simulations of two interacting droplets were performed on the grid schematically illustrated in Fig. 6b. Both droplets were represented by spheres with centers on the  $z$ -axis. The grid points marked with triangles oriented up and down were used to describe kinetics of BZ-reaction in the upper and the lower droplet respectively. The bilayer was represented by a layer of grid points equally distant from centers of droplets. Each of the points representing the bilayer (the black dots in Fig. 6) has neighbors belonging to each of the droplets. We considered excitation pulses that have rotational symmetry along the  $z$ -axis. Within such approximation the problem becomes two-dimensional which speeds up the calculations.

For simulations we assumed that concentrations in the upper droplet were:  $[H_2SO_4] = 0.6M$ ,  $[NaBrO_3] = 0.195M$ ,  $[MA] = 0.175M$ ,  $[KBr] = 0.06M$ , and the catalyst  $0.0017M$ . In the lower droplet they were :  $[H_2SO_4] = 0.6M$ ,  $[NaBrO_3] = 0.45M$ ,  $[MA] = 0.35M$ ,  $[KBr] = 0.06M$ , the catalyst  $0.0017M$ . As in Fig. 5 the frequency of oscillations in the lower droplet is larger than in the upper one. For the considered concentrations the two variable model with re-fitted parameters predicts that separated droplets oscillate with frequencies  $20.5sec$  and  $155sec$ , which agrees with the experiment. If droplets are interacting than oscillations of the lower droplets periodically excite the upper one. The experiment showed a stable 2:1 frequency ratio between excitations arriving at the bilayer and those crossing it. The calculations for interacting droplets were performed in cylindrical coordinates with  $\Delta_t = 1. * 10^{-6}sec$  and equal steps in the z-direction and along the radius  $\Delta_z = \Delta_r = 0.0015mm$ . The radius of spheres representing droplets was 50 grid distances. The bilayer was formed by an array of 22 grid points. Although the diameters of considered droplets was smaller than those in experiments it does not affect simulations, because we considered no flow boundary condition between droplets and hydrocarbons outside. The results of simulations have shown that within the assumptions listed above the interaction between droplets is very strong and each excitation of the lower droplet generates an excitation of the upper one. Thus, in order to describe the observed behavior the amount of transmitted activator should be reduced.

In the FKN model [26] we find reaction:



that describes degradation of activator if no other reagents are present. We can consider this process and describe the time evolution of activator in the lipid bilayer with the equation:

$$\frac{\partial x}{\partial t} = -K_d x^2 + D_x \Delta x \quad (9)$$

and in the reduced variables  $K_d = \epsilon_2 h_0$ .

The numerical simulations have shown that if  $\epsilon_2 = 6700$  then each excitation of the lower droplet still generates an excitation of the upper one. Therefore the mechanism of activator degeneration based on reaction (8) gives too slow decay. Full synchronization between oscillations in droplets was still observed for  $K_d = 10^6$ .

If we assume that some small ions ( $H^+$ ,  $Br^-$ ) can penetrate the lipid bilayer then other reactions can lead to degradation of  $HBrO_2$  inside it. Having no information on which reagents diffuse into the double layer and what are their concentrations we described degradation of  $HBrO_2$  as a simple decay process with the unknown rate  $K$ :

$$\frac{\partial x}{\partial t} = -Kx + D_x \Delta x \quad (10)$$

The value of  $K$  can be fitted from numerical simulations of systems studied in experiments. In numerical simulations for selected concentrations of reagents we observed that the lower droplet, where oscillations are frequent, strongly influenced slowly oscillating upper droplet, but the reversible interaction was

not recorded. We measured the firing number defined as the ratio between the number of excitations in the upper droplet and the number of excitations that arrived from the lower one. Fig.7 illustrates the firing number as a function of the activator decay rate  $K$ . The dependence has a typical devil staircase like form [25]. It is quite surprising that the range of  $K$  in which for the considered concentrations 2:1 frequency transformation is observed is limited to  $[2200sec^{-1}, 4300sec^{-1}]$ . For  $K > 50000sec^{-1}$  oscillations in droplets are not synchronized. The time evolution of  $z$  in the lower and upper droplets for a few selected values of activator decay rate is shown in Fig. 8. The solid curve shows  $z(t)$  at the point on the symmetry axis 16 grid distances above the center of the lower droplet. In this droplet  $z(t)$  was the same for all considered values of  $K$ . All dashed lines plot  $z(t)$  at the point on the symmetry axis 16 grid distances below the center of the upper droplet. The curves corresponding to different values of  $K$  are artificially shifted by a constant, to make them readable. For the considered values of  $K$  we observe different firing numbers. They change from 0.25 for  $K = 7500sec^{-1}$  to 0.5 for  $K = 3500sec^{-1}$ .

The estimated value of  $K$  (here around  $3500sec^{-1}$ ) describes the degeneration of activator that should be taken into account to obtain the transformation of frequency observed in experiments. However, if activator migration through lipids is slower than, assuming the same value of  $K$ , a larger amount of activator will disappear. Therefore, the fitted value of  $K$  is related to the diffusion coefficient of activator in the bilayer. If  $D_l \neq D_x$ , as assumed above, then the value of  $K$  is different.

Numerical simulations indicate that relatively small size of droplets as-

sumed in calculations does not have quantitative influence on the results. Fig. 9 compares  $z(t)$  for droplets with diameters  $r = 50$  and  $r = 100$  grid points. The decay rate used was  $K = 5500\text{sec}^{-1}$ . The upper solid line shows  $z(t)$  in the lower droplet, at the point  $r/6$  above the center. The oscillations in the lower droplet were uniform so the results for  $r = 50$  and  $r = 100$  are identical. In both cases BZ-solution oscillated with the period  $20.5\text{sec}$ . Two lower lines present  $z(t)$  at the point  $r/6$  below the center of the upper droplet. The solid and the dashed lines show results for droplets with diameters  $r = 50$  and  $r = 100$  respectively. If a droplet with selected concentrations does not interact with the other then the period of oscillations is  $155\text{sec}$ . Here oscillations in the upper droplet are more frequent because they are forced by oscillations of the lower one. For both radii:  $r = 50$  and  $r = 100$  every third oscillation of the lower droplet excites the upper one. Therefore, the firing number does not depend on both droplet radius and the size of contact layer. Moreover, due to a large difference in periods of oscillations in separated droplets, the calculated firing number does not depend on the initial states of droplets. The oscillations of the upper droplet are nonhomogeneous. An excitation originates at the point of contact and expands over the droplet. It can be noticed that  $z(t)$  for  $r = 100$  is delayed if compared with  $z(t)$  for  $r = 50$ . The delay just reflects longer distance between the separating bilayer and the observation point inside the larger droplet.

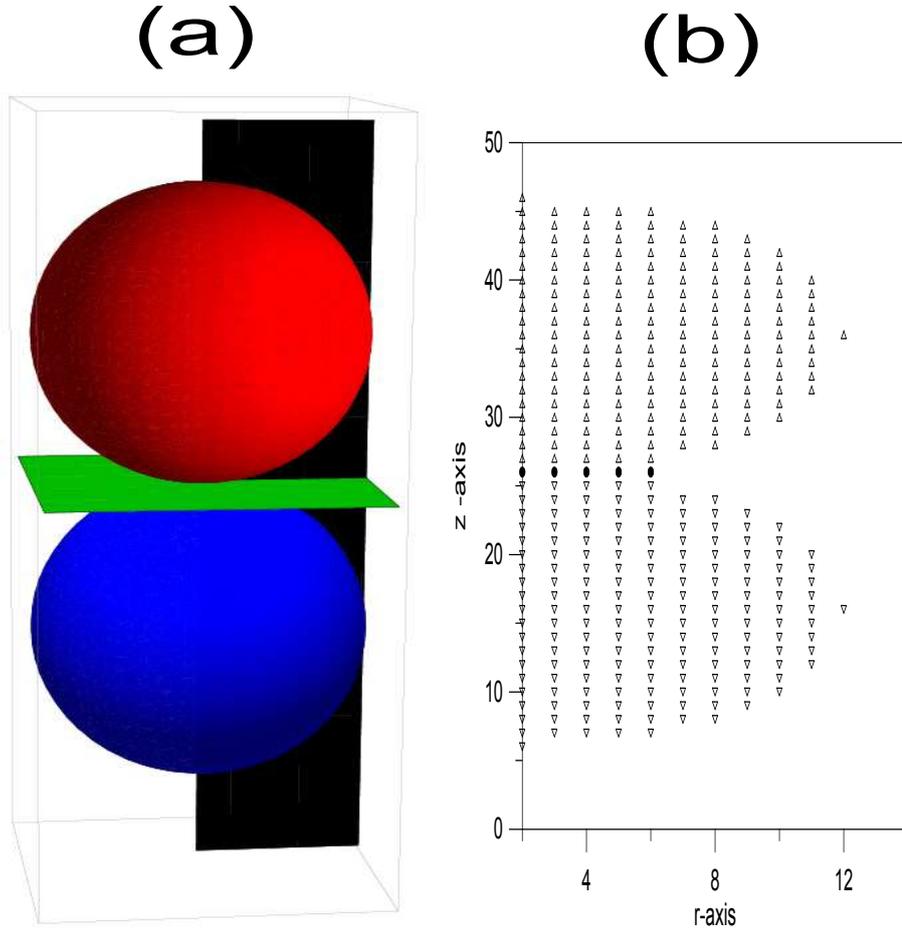


Figure 6: The geometry of two droplets considered in numerical simulations. The horizontal plane separating droplets contains points representing the lipid bilayer. Fig. 6b illustrates a simplified, two-dimensional grid used in simulations of interacting droplets. Triangles oriented up and down correspond to the gridpoints representing the upper and lower droplet. The black dots correspond to the lipid bilayer.

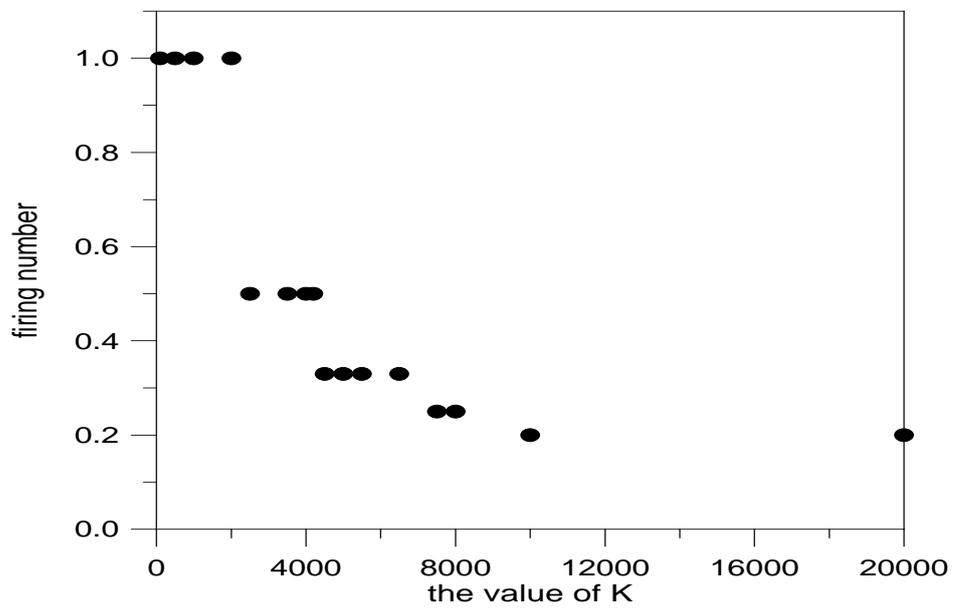


Figure 7: The firing number as the function of the activator decay rate  $K$ .

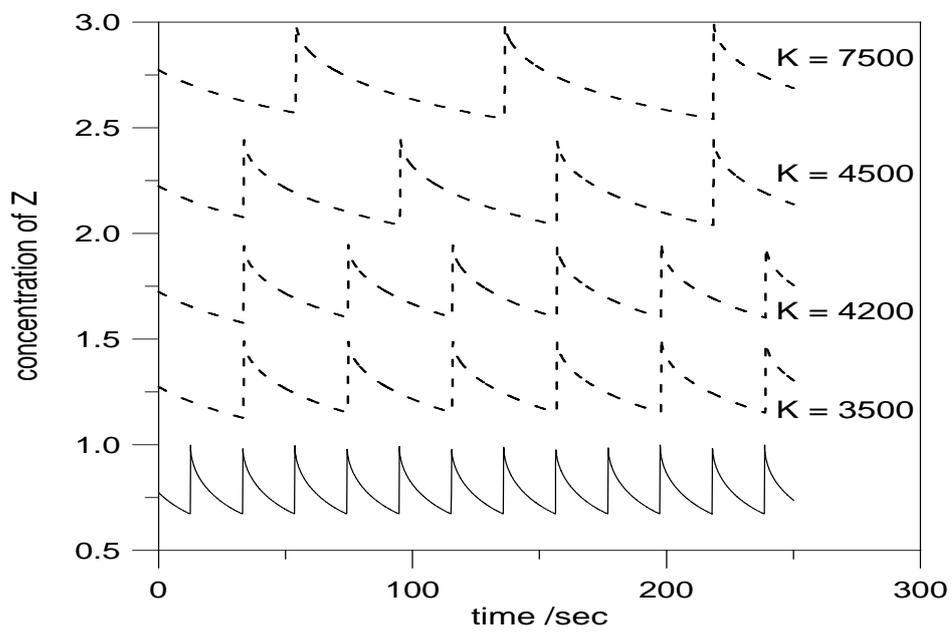


Figure 8: The comparison of  $z(t)$  in the lower droplet (the solid line) and in the upper droplet (the dashed line) for a few values of the activator decay rate  $K$ . The values of  $z(t)$  in the upper droplet are shifted up by a constant factor.

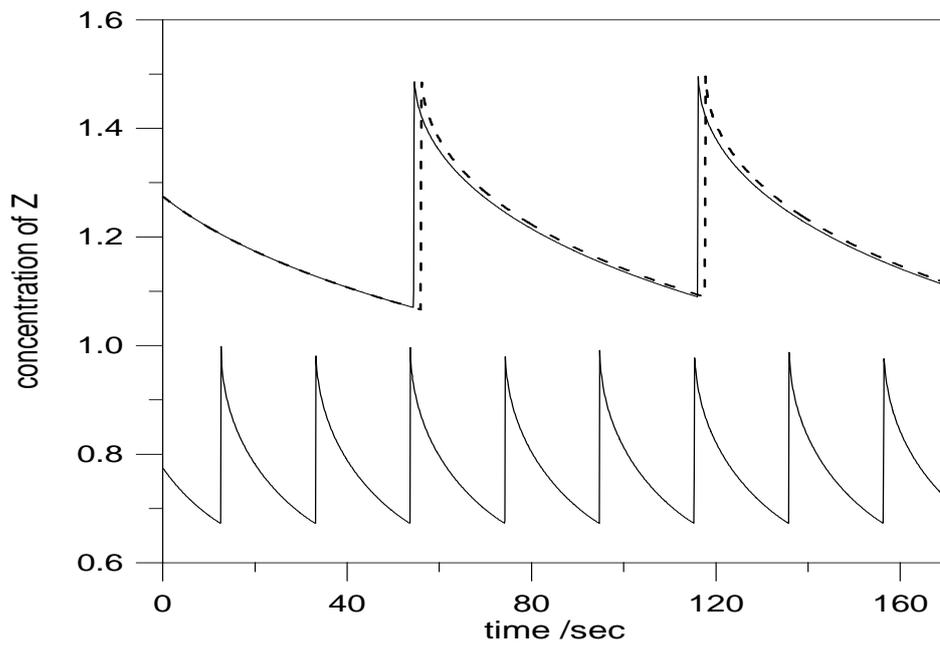


Figure 9: The comparison of  $z(t)$  for droplets with diameters  $r = 50$  and  $r = 100$  grid points.  $K = 5500sec^{-1}$ . The solid line at the bottom shows  $z(t)$  in the lower droplet. The upper solid and dashed lines present  $z(t)$  in the upper droplets with diameters  $r = 50$  and  $r = 100$  respectively.

### 3 The stable modes of coupled oscillations in two identical droplets

The model described in the previous section can be applied to find the stable modes of two coupled BZ-droplets. In coupled nonlinear oscillators one can expect synchronization in both in-phase and anti-phase modes [27]. These modes can represent bits 0 and 1, so a coupled pair of droplets can be used as a memory element in a droplet-based unconventional computer. In numerical study on mode stability we considered two identical droplets represented by spheres with radius  $r = 50$  grid points. The location of droplets were the same as shown in Fig. 7. Concentrations of BZ-reagents inside droplets were identical and equal to:  $[H_2SO_4] = 0.45M$ ,  $[NaBrO_3] = 0.45M$ ,  $[MA] = 0.35M$ ,  $[KBr] = 0.06M$ , and the catalyst  $0.0017M$ . For these concentrations of reagents the calculated period of oscillations is  $T = 35.1sec$ . At the beginning of simulations we assumed uniform concentrations of  $x$  and  $z$  in each droplet. We considered initial concentrations  $(x_0, z_0)$  that corresponded to points on the limit cycle. The droplets were initialized at different points of the cycle. The phase distance between points on the limit cycle can be measured as the time  $t_0$  within which the system evolves between these points. Presenting the results we use  $t_0/T$  as the measure of distance. We studied the distance between concentrations at droplet centers after one period of oscillations  $T$  as the function the initial distance. The results for  $K = 4200sec^{-1}$  are illustrated on Fig. 10. If the distance after time  $T$  is smaller than the initial one then oscillations in droplets finally synchronize in stable in-phase mode. For the considered values of parameters we ob-

served such synchronization if the absolute value of initial phase difference is smaller than  $0.487T$ . It is quite interesting that for majority of initial configurations in this range the stable phase difference is not 0, but has a small value around 0.01. It is because the stable oscillation mode is represented by homogeneous oscillations in one of the droplets that activate excitations in the other. The observed difference in phases corresponds to the time of excitation propagation from the bilayer where excitations appear to the center of droplet where the phase is measured. If the initial phase distance is smaller than  $0.01T$  it remains stable during the evolution. On the other hand if the absolute value of initial phase difference is larger than  $0.49T$  than it does not change after the period. It means that in a narrow range of initial conditions the perturbations generated by excitations arriving from a firing droplet are too small to change the state of the other. Therefore anti-phase oscillations in the considered droplets are stable. Both types of evolution are illustrated in Fig. 11. The solid and dashed lines illustrate  $z(t)$  in the lower and the upper droplets respectively. For difference is phases smaller than  $17.1sec$  oscillations synchronize in in-phase mode within a single cycle. If the difference is larger than  $17.2sec$  (the lines at the bottom) the anti-phase mode of oscillations remains stable. We conclude that if  $K = 4200sec^{-1}$  then a system of two coupled droplets can play a role of memory with states corresponding to in-phase and to anti-phase oscillations, although the region of anti-phase oscillations is very narrow. In the ideal memory based on two interacting droplets the basins of attraction for each of the modes should be similar. The model presented above can be used to scan the space of reagent concentrations for such system. The precise estimation of  $K$  in the model

is very important for studies on the stable modes of interacting droplets, because small changes in  $K$  can lead to qualitative changes in the character of evolution. For smaller values of  $K$  the concentration of transmitted activator increases and the basin of attraction for in-phase synchronization becomes larger. Numerical simulations of BZ-droplets with concentrations described above and the activator degeneration rate  $K = 3000\text{sec}^{-1}$ , which also explains 2:1 frequency transformation in the considered experiment, do not show a stable anti-phase mode. Additional experiments on frequency transformation for droplets with different concentrations of reagents should allow for better estimation of  $K$ .

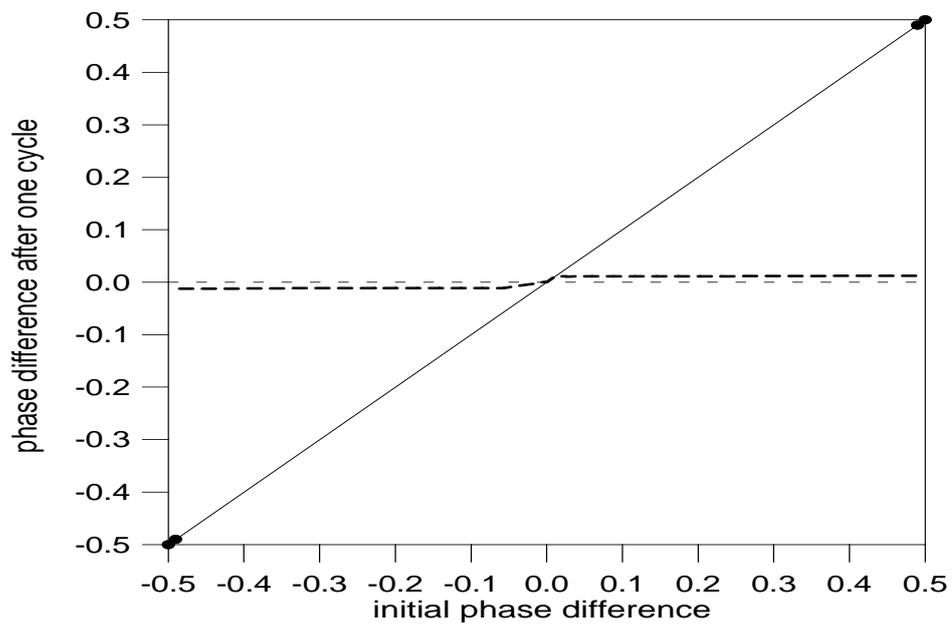


Figure 10: The thick dashed line and symbols at corners show the phase distance between concentrations in the droplet centers measured after time  $T$  as the function of their initial distance. The thin solid line describes the case when the distance after a period is the same as the initial distance. The thin dashed line marks the position of zero.

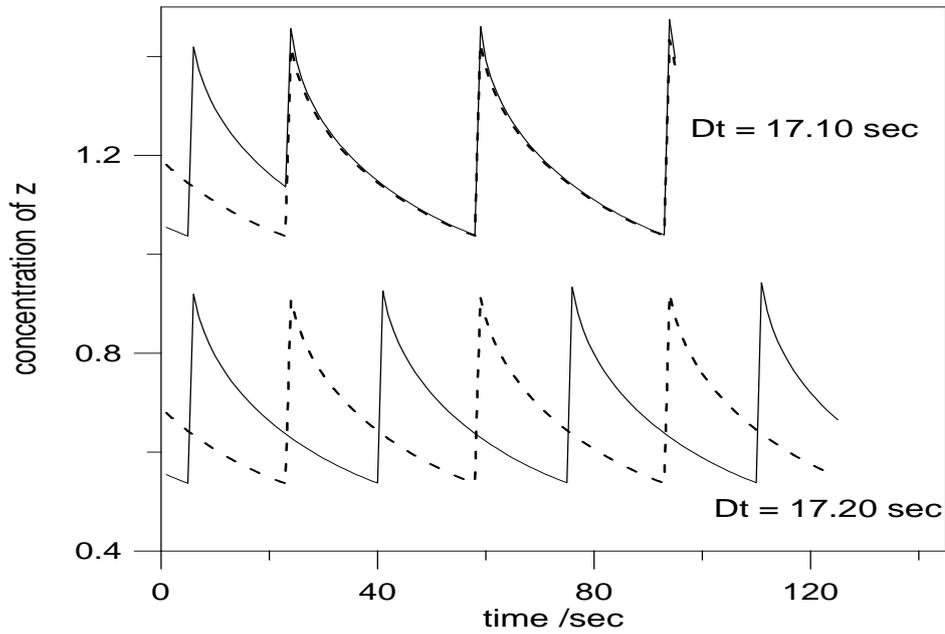


Figure 11:  $z(t)$  for two identical interacting droplets with diameters  $r = 50$ . The solid and dashed lines show  $z(t)$  in the lower and the upper droplets respectively. The initial phase difference between upper and lower droplets is given next to the lines,  $K = 4200\text{sec}^{-1}$ . For the phase difference smaller than  $17.1\text{sec}$  oscillations synchronize in the in-phase mode within a single cycle. If the difference is large it does not change in time and the anti-phase mode of oscillations remains stable (the bottom lines).

## 4 Conclusions

In this paper we generalized previously introduced simple two variable model of BZ reaction kinetics including diffusion of activator and the mechanism of communication between droplets in contact. We estimated the values of model parameters such that numerical calculations based on the model agree with experimental studies on excitation propagation within a droplet and transformation of frequency on the lipid bilayer separating droplets. We think that the considered model can be used in future investigation on information processing operations that can be executed by a medium composed of BZ-droplets. The model contains just two variables and in intensive numerical simulations of spatio-temporal evolution it is faster than multi variable models. This is important because some information processing operations, like for example unidirectional propagation of excitations [13] between droplets, are related with sizes and geometry of contacts. Reaction-diffusion equations describing 3-dimensional medium have to be solved to simulate such behavior. It is also important that the model, unlike commonly used Oregonator [17], can be easily adopted to describe quantitatively an experiment with typical concentrations of reagents.

We applied the model to find the stable modes of oscillations in two coupled droplets containing the same mixture of reagents. The results strongly depend on selected value of  $K$ . For  $K = 4200\text{sec}^{-1}$  that explains 2:1 frequency transformation observed in experiment the model have shown that both in-phase and anti-phase oscillations can be stable, although the region where the anti-phase oscillations are stabile is very small. The stability of

both phases indicates that a pair of oscillating droplets can be used as a memory with 1 bit capacity.

More complex four variable model of BZ-droplet kinetics have been considered in a number of recent papers published by Epstein group [15, 16]. However the droplets in those experiments are much smaller than ours, so the spatiotemporal effects inside a droplet can be neglected. Moreover, the distance between droplets in Epstein group experiments was larger (around  $100\mu m$ ) than that between our droplets in contact. It was discovered that for distant droplets the inhibitory coupling with  $Br_2$  molecules diffusing in the oil phase was dominant and resulted in anti-phase synchronization. In our experiments communication between droplets results from excitatory coupling and in-phase synchronization prevails. However, the model described in the paper shows that even with excitatory coupling anti-phase synchronization can be stable.

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