Abstracts

Energy and Laplacian on Hanoi-type fractal quantum graphs

Patricia Alonso-Ruiz
(Ulm University, Germany)

We introduce fractal quantum graphs, which generalize classical quantum graphs and also include certain non-self similar fractals. We focus on the construction of a Laplacian for so-called Hanoi-type fractal quantum graphs by means of resistance forms and discuss the spectral asymptotics of the obtained Laplacian associated with a family of weakly self similar measures. This is joint work with D.Kelleher and A.Teplyaev from the University of Connecticut.

Harmonizable Fractional Stable Fields: Local Nondeterminism and Joint Continuity of the Local Times

Antoine Ayache
(Lille University of Science and Technology, France)

By using a wavelet method reminiscent of that in the article (Ayache, Shieh and Xiao 2011, AIHP), we prove that, for every $\alpha \in (0, 2)$, the N-parameter harmonizable fractional α-stable field (HFαSF) is locally nondeterministic. When $0 < \alpha < 1$, this solves an open problem in the paper (Nolan 1989, PTRF), also, it allows us, for all $\alpha \in (0, 2)$, to establish the joint continuity of the local times of an (N, d)-HFαSF and to obtain new results concerning its sample paths.

This is a joint work with Yimin Xiao (Michigan State University, USA).

$1/f$ noise and order patterns

Christoph Bandt
(University of Greifswald, Germany)

$1/f$ noise is a stationary process with a mild form of self-similarity. It has been observed in electronic circuits, heart and brain data, internet traffic, stock prices, music etc. but the theory of such noises has still to be developed. A universal mechanism was expected behind all the different phenomena, and a lot of models have been suggested, the most prominent being fractional Gaussian noise.

We study another mild form of self-similarity of stationary processes given by equality of distributions of order patterns. The spectrum of order patterns provides a lot of detail in the low frequency range and allows to distinguish many 1/f noises. The differences show that the processes are generated by different mechanisms.

We give a simple stochastic model which generates a number of different $1/f$ noises and admits a physical interpretation. Since some of the experimental time series show stronger self-similarity than the best theoretical models, there are challenging problems.

On the dimension of the graph of the classical Weierstrass function

Krzysztof Barański
(University of Warsaw, Poland)

We examine dimension of the graph of the famous Weierstrass non-differentiable function

$$W_{\lambda,b}(x) = \sum_{n=0}^{\infty} \lambda^n \cos(2\pi b^n x)$$

for an integer $b \geq 2$ and $1/b < \lambda < 1$. We prove that for every b there exists $\lambda_b \in (1/b, 1)$ such that the Hausdorff dimension of the graph of $W_{\lambda,b}$ is equal to $D = 2 + \frac{\log \lambda}{\log b}$ for every $\lambda \in (\lambda_b, 1)$. We also show that the dimension is equal to D for almost every λ on some larger interval. Moreover, we prove that the Hausdorff dimension of the graph of the function

$$f(x) = \sum_{n=0}^{\infty} \lambda^n \phi(b^n x)$$

for an integer $b \geq 2$ and $1/b < \lambda < 1$ is equal to D for a typical $\mathbb{Z}$-periodic C^3 function ϕ.

This is a joint work with Balázs Bárány and Julia Romanowska.

On invariant measures of IFSs with place-dependent probabilities and the Blackwell measure

Balazs Barany
(Budapest University of Technology and Economics, Hungary)

In this talk we study a family of invariant measures of parametrized iterated function systems where the corresponding probabilities are place-dependent. We show that the Hausdorff dimension of the measure is equal to Entropy/Lyapunov exponent whenever it is less than 1 and the measure is absolute continuous w.r.t. the Lebesgue measure if Entropy /Lyapunov exponent is strictly greater than 1 for Lebesgue almost every parameters.

As an application, we study the Blackwell measure, which play an important role in information theory and the study of Lyapunov exponents. We show that for a suitable set of parameter values the Blackwell measure is absolutely continuous for almost every parameter in the case of binary symmetric channels.

Symbolic IFS theory, fast basins and fractal manifolds

Michael Barnsley
(Australian National University, Australia)

I will describe a generalised notion of analytic continuation that applies to self-analytic sets. A subset of R2 is called self-analytic when it is the attractor of an analytic IFS. I will describe some properties of an associated topological invariant called a fractal manifold.

Percolation on a Gibbs weighted tree

Julien Barral
(University of Paris 13, France)

Given a Gibbs measure on the symbolic space over a finite alphabet, we use a percolation process to derive a random capacities with a rich multifractal analysis; this construction naturally yields new models of lacunary wavelet series with controlled multifractal nature, as well a energy models with two phase transitions (this is a joint work with Stéphane Seuret).

On random fractals with infinite branching: definition, measurability, dimensions

Artemii Berlinkov
(-, Russia)

We discuss the definition and measurability questions of random fractals with infinite branching, their lower, upper Minkowski dimensions as well as packing dimension.

Minkowski content and curvature measures of random self-conformal fractals

Tilman Bohl
(University of Jena, Germany)

We describe stochastically self-conformal fractals in terms of volume and curvature. This leads to the (Cesaro-averaged) Minkowski content, its local measure version, and to fractal Lipschitz-Killing curvature-direction measures. The principal tool is a skew product dynamical system given by local covariance. See [1, 2] for the deterministic case.

An conformal iterated function system (CIFS) is a finite system of contractions ϕ_i whose differentials preserve angles. Self-similarity is a "rigid" special case. A random conformal iterated function system (RCIFS) is a finite collection of CIFS, whose member contractions are called $\phi_{i,a}$.

The deterministic case $a \equiv 1$ gives rise to the self-conformal set $F \subseteq \mathbb{R}^d$, $F = \bigcup_{i\in I} \phi_{i,a} F$. In the random case, a random subsystem $a \in A$ is chosen independently at each step of the construction. Thus the random attractor set satisfies

$$F = F(a_1, a_2, \dots) = \bigcup_i \phi_{i,a_1} F(a_2, a_3, \dots)$$

along an iid sequence $(a_1, a_2, \dots) \in A^{\mathbb{N}}$ of CIFSs.

F is "invisible", i.e., has trivial Lebesgue measure $\mathcal{L}(F)$. So we approximate F with an r-thin "coating" (parallel set) $F_r = \{x : \operatorname{dist}(x, F) \le r\}$ and study its volume $\mathcal{L}(F_r)$. Rescaling suitably with r^{D-d} prevents that $r^{D-d}\mathcal{L}(F_r)$ vanishes trivially as $r \to 0$. Cesaro-averaging makes sure a limit exists. Under weak assumptions, we prove the average total Minkowski content

$$\lim_{\epsilon\to 0} \frac{1}{|\ln \epsilon|} \int_\epsilon^1 r^{D-d} \mathcal{L}(F_r) \frac{d\epsilon}{\epsilon} = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^0 e^{t(D-d)} \mathcal{L}(F_{e^t})\, dt \tag{1}$$

of the random set F converges non-trivially. It can be localized as a measure. There is an integral formula.

Next, we consider geometrical quantities other than volume. You can the hear shape of a differentiable drum: it is well-known that spectral asymptotics determine the curvature and length of its (smooth) boundary. This fact motivates a curvature description of fractals. Curvature involves second order "coordinate derivatives". To grasp curvature intuitively, pretend F_r is a subset of the plane and bounded by a smooth curve ∂F_r. The principal curvature $\kappa(x)$ of ∂F_r is the density function of the Gauss curvature measure. More precisely, geometric measure theory defines Federer curvature measures [4, 3]. Fractal curvature arises when we replace the Lebesgue measure $\mathcal{L}$ with curvature in Eq. (1). We prove the (modified) limit converges in probability under an integrability condition. Again, it can be localized, and there is an integral formula.

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Averages along the squares on the torus

Zoltan Buczolich
(Eötvös Loránd University, Hungary)

We show that for any $x, \alpha \in \mathbb{T}$, $\alpha \notin \mathbb{Q}$ there exist $f \in L^1(\mathbb{T})$, $f \geq 0$ such that the averages

$$(\star) \qquad \frac{1}{N}\sum_{n=1}^{N} f(y + nx + n^2\alpha)$$

diverge for a.e. y. By Birkhoff's Ergodic Theorem applied on $\mathbb{T}^2$ for the transformation $(x, y) \mapsto (x + \alpha, y + 2x + \alpha)$ for almost every $x \in \mathbb{T}$ the averages $(\star)$ converge for a.e. y. We show that given $\alpha \notin \mathbb{Q}$ one can find $f \in L^1(\mathbb{T})$ for which the set $D_{\alpha,f} \stackrel{\text{def}}{=} \{x \in \mathbb{T} : (\star)$ diverges for a.e. y as $N \to \infty\}$ is of Hausdorff dimension one. We also show that for a polynomial $p(n)$ of degree two with integer coefficients the sequence $p(n)$ is universally L^1-bad. This answers a question raised by J-P. Conze.

Reinforcement problems for fractal structures

Raffaela Capitanelli
(Sapienza University of Rome, Italy)

Reinforcement problems in the classical setting of regular domains was widely studied in conjunction with numerous applications. In this talk, I present some results on reinforcement problems for fractal structures.

Projections of random covering sets

Changhao Chen
(University of Oulu, Finland)

Joint work with: Henna Koivusalo, Bing Li, Ville Suomala

Given a sequence of independent random variables (ξ_n), uniformly distributed on the torus $\mathbb{T}^d$, and a sequence of subsets of the torus, (g_n), random covering set E is the set of infinitely often covered points,

$$E = \limsup_{n\to\infty}(\xi_n + g_n) = \bigcap_{k=1}^{\infty}\bigcup_{n=k}^{\infty}(\xi_n + g_n).$$

Here we interpret $\xi_n + g_n \subset \mathbb{T}^d$.

We show that, almost surely, the Hausdorff dimension s_0 of a random covering set is preserved under all orthogonal projections to linear subspaces with dimension $k > s_0$. The result holds for random covering sets with a generating sequence of ball-like sets, and is obtained by investigating orthogonal projections of a class of random Cantor sets.

Maxima and entropic repulsion of Gaussian free fields on fractal-like graphs

Joe Chen
(University of Connecticut, United States)

The Gaussian free field on an (infinite) graph is a centered Gaussian process whose covariance is the Green's function for simple random walk on the graph. As a model random surface which favors minimization of "gradient", one would like to study its stochastic geometry, such as its maximal height. A related problem, known as "entropic repulsion," concerns the height of the field conditioned upon it being positive everywhere, as if a "hard wall" were imposed at zero height.

In this talk I will explain how the maxima problem and the entropic repulsion problem are related, and give leading-order asymptotic height of the free field on various types of fractal graphs. These extend previous results on $\mathbb{Z}^d$, and rely crucially on the Mosco convergence of Dirichlet forms, as well as a coarse-graining procedure. As an application, we also obtain asymptotics of cover times of random walk on high-dimensional Sierpinski carpet graphs.

Heat kernels for non-symmetric stable-like operators

Zhen-Qing Chen
(University of Washington, United States)

Let $d \geq 1$ and $\alpha \in (0, 2)$. Consider the following non-local and non-symmetric Lévy-type operator on $\mathbb{R}^d$:

$$\mathcal{L}_\alpha^\kappa f(x) := \text{p.v.} \int_{\mathbb{R}^d} (f(x+z) - f(x)) \frac{\kappa(x,z)}{|z|^{d+\alpha}} dz,$$

where $0 < \kappa_0 \leq \kappa(x,z) \leq \kappa_1$, $\kappa(x,z) = \kappa(x,-z)$, and $|\kappa(x,z) - \kappa(y,z)| \leq \kappa_2 |x-y|^\beta$ for some $\beta \in (0,1)$. Using Levi's method, we construct the fundamental solution (also called heat kernel) $p_\alpha^\kappa(t,x,y)$ of $\mathcal{L}_\alpha^\kappa$, and establish its sharp two-sided estimates as well as its fractional derivative and gradient estimates. We also show that $p_\alpha^\kappa(t,x,y)$ is jointly Hölder continuous in (t,x). The lower bound heat kernel estimate is obtained by using a probabilistic argument. The fundamental solution of L_α^κ gives rise a Feller process $\{X, \mathbb{P}_x, x \in \mathbb{R}^d\}$ on $\mathbb{R}^d$. We determine the Lévy system of X and show that $\mathbb{P}_x$ solves the martingale problem for $(\mathcal{L}_\alpha^\kappa, C_b^2(\mathbb{R}^d))$. As an application of the above result, we derive sharp two-sided estimates for the transition density of the solution of $dX_t = A(X_{t-})dY_t$, where Y is a (rotationally) symmetric stable process on $\mathbb{R}^d$ and $A(x)$ is a Hölder continuous $d \times d$ matrix-valued function on $\mathbb{R}^d$ that is uniformly elliptic and bounded.

A multifractal-based climate analysis

Adrien Deliège
(University of Liège, Belgium)

This work consists of a multifractal analysis of surface air temperature in Western and Eastern Europe. The wavelet leaders method enables us to exhibit the monofractal nature of these signals. Then, we define a climate classification relying on the Zygmund-Hölder spaces and show that it matches the Köppen classification. By doing so, we evidence the existing relation between the regularity of the temperature data (obtained through functional spaces) and the type of climate. Finally, we give a climatic interpretation of these results.

Bounds for the critical value in fractal percolation

Henk Don
(Radboud University Nijmegen, Netherlands)

Over the years several bounds have been given for the critical probability in fractal percolation. I will briefly review the model and discuss some of these bounds. An idea to find sharper bounds will be presented together with some results.

Groups of Homeomorphisms of Cantor Sets

Casey Donoven
(University of St Andrews, United Kingdom)

Many interesting homeomorphisms from self-similar Cantor sets to themselves arise by exploiting their similar, symmetrical, and disconnected cylinders. I will discuss the Thompson-like groups, $V_n(G)$, and distinguish an isomorphism class through orbit dynamics and transducers.

A new multifractal formalism based on wavelet leaders: detection of non concave and non increasing spectra (Part I)

Céline Esser
(University of Liège, Belgium)

Multifractal analysis is concerned with the study of very irregular signals. For such functions, the pointwise regularity may change widely from a point to another. Therefore, it is more interesting to determine the spectrum of singularities of the signal, which is the Hausdorff dimension of the set of points which have the same Hölder exponent. The spectrum of singularities of many mathematical functions can be determined directly from its definition. However, for many real-life signals, the numerical determination of their Hölder regularity is not feasible. Therefore, one cannot expect to have a direct access to their spectrum of singularities and one has to find an indirect way to compute it.

A multifractal formalism is a formula which is expected to yield the spectrum of singularities from quantities which are numerically computable. Several multifractal formalisms based on the wavelet coefficients of a signal have been proposed to estimate its spectrum. The most widespread of these formulas is the so-called thermodynamic multifractal formalism, based on the Frish-Parisi conjecture [5]. This formalism presents two drawbacks: it can hold only for spectra that are concave and it can yield only the increasing part of the spectrum. This first problem can be avoided using S^ν spaces [1, 4].

The second one can be avoided using a formalism based on wavelet leaders of the signal [6].

In this talk, we propose a new multifractal formalism, based on a generalization of the S^ν spaces using wavelet leaders. It allows to detect non concave and non increasing spectra [2, 3]. An implementation of this method is presented in the talk *A new multifractal formalism based on wavelet leaders: detection of non concave and non increasing spectra (Part II)* of T. Kleyntssens.

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Dirichlet eigenvalues of a measure geometric Laplacian through a modified trigonometric function and characteristic polynomials

Roland J. Etienne

(University of Siegen, Germany)

We investigate the Dirichlet eigenvalues of the measure geometric Laplacian $\Delta^{\mu,x} = -\frac{d}{d\mu}\frac{d}{dx}$ acting on $L_2([0,1],\mu)$ where μ is a self-similar measure with support K. We define an analogue of the sine function through a power series such that its squared zeroes give the eigenvalues of $\Delta^{\mu,x}$. Furthermore, we will give a physical interpretation of this operator, thereby obtaining approximations to the modified sine function by characteristic polynomials of matrices, reminiscent of an Euler product expansion.

60 Years of Projections

Kenneth Falconer

(University of St Andrews, Scotland)

It is 60 years since the publication of John Marstrand's 1954 paper 'Some fundamental geometrical properties of plane sets of fractional dimension' which included his famous projection theorems. Arguably, this paper marked the start of fractal geometry as we know it today. This talk will celebrate this anniversary with some reflections both on the theorem and its numerous generalisations and specialisations, and on John himself.

Linear images of self-similar sets with no separation condition

Ábel Farkas
(University of St Andrews, United Kingdom)

We investigate how the Hausdorff dimension and measure of a self-similar set $K \subseteq \mathbb{R}^d$ behave under linear images. It turns out that this depends on the nature of the group generated by the orthogonal parts of defining maps of K. We prove our results without assuming any separation condition. We introduce a new method that leads to a similarity dimension-like formula for Hausdorff dimension.

Self-similar subsets of the Cantor set

De-Jun Feng
(Chinese University of Hong Kong, China)

We study the following question raised by Mattila in 1998: what are the self-similar subsets of the middle-third Cantor set C? We prove that any non-singleton self-similar subset of C should have contraction ratios 3^{-n}, where n is an integer. We give a complete characterization of equal-contractive self-similar subsets of C. For the non equal-contractive case, we provide a finite algorithm to determine all the self-similar subsets with pre-given contraction ratios.

This is joint work with Hui Rao and Yang Wang.

Inhomogeneous iterated functions systems

Jonathan Fraser
(University of Warwick, United Kingdom)

Inhomogeneous iterated function systems, introduced by Barnsley and Demko, are a natural generalisation of the classical iterated function system. They have proved useful both in theory and in practise with applications in understanding self-similar sets with overlaps and to image compression. We will revue some structural and dimensional results concerning the attractors of such systems and give some examples illustrating some unexpected behaviour.

Poincaré functional equations, harmonic measures on Julia sets, and fractal zeta functions

Peter Grabner
(Graz University of Technology, Austria)

Iterative functional equations, such as the Poincaré equation and the Böttcher equation have been studied in complex dynamics in order to obtain a deeper understanding of the local behaviour of the iterates of polynomials $P(z)$. The Poincaré functional equation

$$f(\lambda z) = P(f(z)), \quad P'(f(0)) = \lambda$$

for $f(0)$ a fixed point of P provides a local linearisation of the function P around $f(0)$, if $|\lambda| > 1$. Similarly, the Böttcher equation

$$g(z)^d = g(P(z)), \quad d = \deg P$$

provides a normalisation of P around infinity. Combining these two functions provides precise information about the asymptotic behaviour of $f(z)$ for $z \to \infty$ and $f(z) \to \infty$ in some angular region. This can be related to the behaviour of $g(z)$, when z approaches certain points of the Julia set of P. This complements Valiron's classic results about the asymptotic behaviour of the maximum function

$$M(r) = \max_{|z|=r} |f(z)|$$

of f. We could prove a Pommerenke-Levin type estimate for the multiplier λ, if the Julia set is real.

Further interest in Poincaré functions f comes from the fact that the spectrum of the Laplace operator on fractals with spectral decimation can be described in terms of level sets of f. This allows to give an analytic continuation of the ζ-function of this Laplace operator to the whole complex plane. Furthermore, special values of this ζ-function can be computed. The precise study of the asymptotic behaviour of f allows to relate the zero counting of f with the harmonic measure on the Julia set of P.

This talk is based on joint work with Gregory Derfel (Ben Gurion University of the Negev, Beer Sheva) and Fritz Vogl (Vienna University of Technology).

From Fractal Groups to Fractal Sets and Spectra

Rostislav Grigorchuk
(Texas A&M University, United States)

I will begin with a panorama of interaction between group theory and fractal geometry. This will include the notions of self-similar group, branch group, iterated monodromy group, Mealy type automata etc.

On the way to fractals we will deal with the Schreier graphs. Such examples as lamplighter group, Basilica (or $IMG(z^2 - 1)$), Hanoi Towers groups $H^{(k)}$ and other interesting examples of groups will be mentioned. After that I will lend in the world of spectra of groups, graphs and fractals. At the end the (historically) first Julia set and the associated fractal group will be used to illustrate the techniques of computation of spectra based on the ideas of self-similarity, multiparameter approach to the spectral problem, and the generalized Schur complement method.

Spectral asymptotics for random self-similar fractals

Ben Hambly
(University of Oxford, United Kingdom)

It is a classical result due to Weyl that $N(\lambda)$, the eigenvalue counting function for a bounded domain, grows as $C\lambda^{d/2}$ for a known constant C proportional to the volume of the domain. We will discuss the asymptotics of the eigenvalue counting function, partition function and heat content for some random self-similar sets and sets with random fractal boundary. It is known that for simple examples of such sets there is an almost sure leading order Weyl asymptotic, in that with probability one, the eigenvalue counting function satisfies $\lim_{\lambda\to\infty} N(\lambda)\lambda^{-d_s/2} = W$, where d_s is the spectral dimension and W is a random variable. We will investigate the second order behaviour of these functions for some classes of random self-similar sets and show that in certain cases there are central limit theorems for these functions.

Estimation of the functional Hurst's parameter of Linear Multifractional Stable Motion by discrete variations method.

Julien Hamonier
(ENS Lyon, France)

The Linear Multifractional Stable Motion (LMSM) is a real Strictly α-stable stochastic process which was introduced by Stoev and Taqqu in [5, 6] with a view to model some features of traffic traces on telecommunications networks, typically changes in operating regimes and burstiness (the presence of rare but extremely busy periods of activity). This process is obtained by replacing the constant Hurst parameter of the Linear Fractional Stable Motion (LFSM) by a function $H(\cdot)$. Throughout our talk, we will assume that $\alpha \in (1, 2)$, the function $H(\cdot)$ takes values in $(1/\alpha, 1)$ and satisfies an uniform Hölder condition of order $\rho_H \geq \max_{t\in[0,1]} H(t)$ over $[0, 1]$.

In the case of LFSM, the statistical issue of estimation of H has already been studied in several works by using wavelet coefficients or discrete variations of this process. (see e.g [1, 2, 3, 4]).

In the LMSM's case, the problem is harder since the Hurst parameter changes with time and the increments of the process are no longer stationary. Therefore, we propose to estimate $H(t)$ and $\min_{t\in I} H(t)$ (I is a fixed, non-empty, compact interval) thanks to discrete variations of LMSM. *A joint work with Antoine Ayache, University Lille1.*

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Function spaces on h-sets: traces, envelopes, embeddings

Dorothee D. Haroske
(University of Jena, Germany)

We study (weighted) spaces of Sobolev and Besov type with respect to their traces on fractal h-sets. This continues and extends earlier work of Michele Bricchi started in 2002. Recently we could prove that (for fixed integrability parameters) often there appears an alternative, a so-called dichotomy: one can precisely distinguish between these smoothness parameters where the corresponding spaces admit traces, and those where the test functions supported outside the fractal set are dense in that space. Naturally, also weight parameters and the geometry of the underlying sets have strong influence on this phenomenon. Secondly, we shall characterise the quality (in terms of unboundedness) of distributions belonging to such spaces via their growth envelopes. This fairly new tool admits better understanding of the interplay between the geometry of the underlying fractal and the corresponding function spaces. Finally, we collect some results about embeddings of function spaces on h-sets.

The talk is based on joint work with Antonio Caetano (Aveiro/Portugal).

The scaling limit of loop-erased random walks on fractals – the erasing-larger-loop-first model and the uniform spanning trees

Kumiko Hattori
(Tokyo Metropolitan University, Japan)

We study loop-erased random walks on some finite fractals including the pre-Sierpinski gasket. We prove the existence of the scaling limit, and show that the path of the limiting process is almost surely self-avoiding, while having Hausdorff dimension strictly greater than one. This result means that the path has infinitely fine creases, while having no self-intersection.

In erasing loops from a simple random walk, we employ a method of erasing largest-scale loops first and going down step by step to the smallest, which allows us to make effective use of the 'self-similarity' of the fractal lattices. On the Sierpinski gasket our model gives the same loop-erased random walk obtained from the uniform spanning trees studied by Shinoda, Teufl and Wagner.

On Lipschitz maps and dimension

Irmina Herburt
(Warsaw University of Technology, Poland)

Maria Moszyńska and the first author suggested some natural axioms for fractal dimension functions. We discuss the independence of these axioms. In particular, using the Continuum Hypothesis, we associate to each nonempty separable metric space X a non-negative integer $d(X)$ so that the function d is Lipschitz subinvariant, stable under finite unions, $d([0,1]^n) = n$, but still, for some $E \subset [0,1]^3$ we have $d(E) < \dim E$, where $\dim E$ is the topological dimension of E.

1-forms and vector fields on fractals

Michael Hinz
(Bielefeld University, Germany)

We survey some of our recent results on a vector analysis based on Dirichlet forms. They rely on the approach to generalized L_2-differential 1-forms by Cipriani and Sauvageot. It yields a feasible notion of first order derivation on fractal spaces carrying a Dirichlet form, what allows to study quasilinear scalar equations and vector equations (such as magnetic Schrödinger equations or Navier-Stokes systems). In particular, we will explain a new Hodge type theorem for topologically one-dimensional fractals. Our results are joint with Alexander Teplyaev (University of Connecticut).

Dimension of self-similar sets in R^d (with overlaps)

Michael Hochman
(Hebrew University of Jerusalem, Israel)

I will describe some conjectures and recent progress on calculating the dimension of self-similar sets in R^d, in the presence of nontrivial overlaps.

Dynamics of infinitely generated expanding semigroups of rational maps

Johannes Jaerisch
(Osaka University, Japan)

We consider the dynamics of semigroups of rational maps on the Riemann sphere. We prove Bowen's formula for the Hausdorff dimension of the Julia set of a possibly infinitely generated nicely expanding rational semigroup. We give applications to non-hyperbolic rational semigroups by employing the method of inducing. This is a joint work with Hiroki Sumi (Osaka University).

Multifractal analysis based on the p-exponent

Stéphane Jaffard
(Paris 12 Val de Marne University, France)

The purpose of the multifractal analysis of a function f is to determine the Hausdorff dimensions of the sets of points where a pointwise regularity exponent $h_f(x)$ takes a given value H. The corresponding collection of dimensions $d_f(H)$ is usually referred to as the *multifractal spectrum* of f. In applications, these dimensions are estimated as a Legendre transform of averaged quantities, effectively computable on data. If the regularity exponent is the usual Hölder exponent, this procedure only applies to locally bounded functions, an assumption often not met in applications.

Our purpose in this talk is to give an overview of the alternative supplied by using the p-exponent, which can be done as soon as the data locally belong to L^p. We will present both the mathematical developments, but also applications to stochastic processes and real-life data, for which the previous approach based on the Hölder exponent did not work. This is joint work with Patrice Abry, Roberto Leonarduzzi, Stéphane Roux, María Eugenia Torres and Herwig Wendt.

Projections for random self-similar measures and sets

Xiong Jin
(University of Manchester, United Kingdom)

I will talk about the exact-dimensionality and dimension conservation of random cascade measures on self-similar sets, and some generalisations of Hochman and Shmerkin's results on the lower semi-continuity of the dimension of projections. This is joint work with Kenneth Falconer.

Dimensions of random covering sets in torus

Esa Järvenpää
(University of Oulu, Finland)

For a general class of generating sets, we study dimensional properties of random covering sets in d-dimensional torus. This is joint work with De-Jun Feng and Ville Suomala.

Dimensions of random covering sets in torus

Maarit Järvenpää
(University of Oulu, Finland)

For a general class of generating sets, we study dimensional properties of random covering sets in d-dimensional torus. This is joint work with De-Jun Feng and Ville Suomala.

Spectral volume measure via the Dixmier trace for Dirichlet spaces with Weyl type eigenvalue asymptotics

Naotaka Kajino
(Kobe University, Japan)

The purpose of this talk is to present a recent result of the author on an operator theoretic way of looking at Weyl type Laplacian eigenvalue asymptotics for Dirichlet spaces.

The asymptotic behavior of the eigenvalues of a given non-negative self-adjoint operator typically involves a certain "volume" of the state space as the proportionality constant, as is well-known for the Laplacian on Riemannian manifolds. Considering the same operator on an open subset of the state space with Dirichlet boundary condition usually results in the same eigenvalue asymptotics with the volume of the state space replaced by that of the open subset, which could be considered as showing how the operator is "spatially distributed".

There is an alternative, purely operator theoretic way of extracting this "spatial distribution" of the given operator based on the Dixmier trace Tr_ω, which is a trace functional defined on a certain ideal of compact operators on a Hilbert space and is meaningful e.g. for compact non-negative self-adjoint operators whose n-th largest eigenvalue is comparable to $1/n$. For the Laplacian Δ on a d-dimensional Riemannian manifold M, Connes' trace theorem implies that the linear functional $f \mapsto \mathrm{Tr}_\omega(f(-\Delta)^{-d/2})$ coincides with (a constant multiple of) the integral with respect to the Riemannian volume measure of M.

A similar fact was proved also for Laplacians on fractals by Kigami and Lapidus in 2001, but their argument was based on a rather strong assumption and appilicable to only a limited class of finitely ramified self-similar fractals. In fact, as will be presented in the talk, the author has recently realized that such a strong assumption is not necessary and that the same result can be proved for a general Dirichlet space satisfying Weyl type asymptotics for the trace of its associated heat semigroup. This result is applicable to any finitely ramified self-similar fractal, any generalized Sierpiński carpet, random recursive Sierpiński gaskets and the continuum random tree.

The size of sets containing squares centered at every point

Tamás Keleti
(Eötvös Loránd University, Hungary)

By a classical theorem of Bourgain and Marstrand, if a subset of the plane contains a circle centered at every point of [0,1]x[0,1] then the set must have positive Lebesgue measure.

What happens if we replace circles by squares? How small a set can be that contains a square centered at every point of [0,1]x[0,1]?

It is surprisingly easy to construct very small sets with this property: such a set can have even Hausdorff dimension 1.

The Minkowski dimension turns out to be more interesting: we show that the minimal possible Minkowski dimension is 7/4, at least if we allow only axis-parallel squares, and we conjecture that this restriction is superfluous.

This is joint work with Pablo Shmerkin and Dániel Nagy.

From Self Similar groups to intrinsic metrics: Vector analysis for Dirichlet forms on fractals

Daniel Kelleher
(University of Connecticut, United States)

We will discuss the possibility of defining vector analysis for measurable Dirichlet forms (quadratic forms on scalar functions). This vector analysis has applications to the Dirac operator, and the existence of the intrinsic metrics. This construction combines ideas from classical and non-commutative functional analysis and if time permits we shall discuss how this leads to the definition of spectral triples on fractal spaces.

Cheeger's inequality for unbounded graph Laplacians

Matthias Keller
(University of Jena, Germany)

We present a Cheeger inequality for general Laplace operators on graphs. Such an inequality relates an isoperimetric constant to lower bounds on the spectrum of the Laplacian. While until now such a result was only available for the normalized Laplacian - which is always a bounded operator - our results hold for arbitrary weighted graph Laplacians. In particular, this solves a questions raised by Dodziuk/Kendall in 1986. The new ingredient is the concept of intrinsic metrics for non-local Dirichlet spaces due to Frank/Lenz/Wingert. (This is joint work with Frank Bauer and Radoslaw Wojciechowski.)

Measures on Slices and Equidistribution for Sets of beta-expansions.

Tom Kempton
(University of St Andrews, United Kingdom)

Motivated by an equidistribution result of Lindenstrauss, Peres and Schlag, we investigate the problem of disintegrating Hausdorff measure on a self similar set by slicing. This allows us to give conditions under which a typical slice through a self similar set has positive Hausdorff measure.

On the quantization for self-affine measures on Bedford-McMullen carpets

Marc Kesseböhmer
(University of Bremen, Germany)

For a self-affine measure on a Bedford-McMullen carpet we prove that its quantization dimension exists and determine its exact value. Further, we give various sufficient conditions for the corresponding upper and lower quantization coefficient to be both positive and finite. Finally, we compare the quantization dimension with corresponding quantities derived from the multifractal temperature function and show that – different from conformal systems – they in general do not coincide.

Volume doubling property, quasisymmetry and time change of Brownian motion

Jun Kigami
(Kyoto University, Japan)

We consider time changes of the Brownian motions of the Sierpinski carpets including the Euclidean spaces under measures which are not necessarily self-similar. In short, a time change is to put a (singular) density of a medium and locally change the speed of the Brownian motion. First we give a sufficient condition for time change. Then under the volume doubling property to the normalized Hasudorff measure, we show the existence of a metric which is quasisymmetric to the (restriction of) Euclidean metric and under which we have nice upper and lower heat kernel estimates.

A counterexample to the Keleti perimeter to area conjecture

Viktor Kiss
(Eötvös Loránd University, Hungary)

Tamás Keleti proved, that if we take a finite union of unit squares, than the ratio of the perimeter and the area cannot be arbitrarily large. He asked weather the ratio is always at most 4. We show an example where it is greater than 4.

Joint work with Zoltán Vidnyánszky.

A new multifractal formalism based on wavelet leaders: detection of non concave and non increasing spectra (Part II)

Thomas Kleyntssens
(University of Liège, Belgium)

This talk follows *A new multifractal formalism based on wavelet leaders: detection of non concave and non increasing spectra (Part I)* given by Céline Esser. To characterize very irregular functions, it is interesting to examine their spectrum of singularities, i.e. "the size" of the set of points which share the same pointwise irregularity; by size, one means the Hausdorff dimension. For real-life signals, it is impossible to compute the spectrum of singularities by using its definition. A multifractal formalism is used to approximate this spectrum. Currently, there exist several formalisms ([4, 5, 1, 2]).

We present a new multifractal formalism for non concave and non increasing spectra. The approximation of the spectrum of f is given by

$$\widetilde{\nu}_f(\alpha) = \begin{cases} \lim\limits_{\epsilon\to 0^+} \left(\limsup\limits_{j\to+\infty} \left(\dfrac{\log(\#\{k : d_{j,k} \geq C2^{-(\alpha+\epsilon)j}\})}{\log(2^j)} \right) \right) & \text{if } \alpha \leq \beta \\ \lim\limits_{\epsilon\to 0^+} \left(\limsup\limits_{j\to+\infty} \left(\dfrac{\log(\#\{k : d_{j,k} < C2^{-(\alpha-\epsilon)j}\})}{\log(2^j)} \right) \right) & \text{otherwise} \end{cases}$$

where $(d_{j,k})_{j\in\mathbb{N},k\in\{0,\ldots,2^j-1\}}$ are the wavelet leaders of f, β is the smallest positive real such that $\widetilde{\nu}_f(\beta) = 1$ and C is a positive constant.

In practice, one has to avoid the concept of limit and to deal with finite size effects. For example, in theory, the constant C appearing in the definition of $\widetilde{\nu}_f$ is arbitrary, but in practice one can only calculate a finite number of wavelet leaders. So, if the constant is too small or too big, we count too many coefficients or not enough. The choice of this constant becomes capital to determine the correct spectrum. In this talk, I will present an implementation of this formalism and I will illustrate it numerically on theoretical functions.

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A shrinking target problem for typical affine sets

Henna Koivusalo
(University of York, United Kingdom)

A shrinking target problem is the study of the set of points recurring to a sequence of targets infinitely often under iteration of a dynamics. To put it more precisely, for a dynamical system (X, T) and a nested sequence of target sets $U_1 \supset U_2 \supset \dots$, it is the study of the set

$$\{x \in X \mid T^n(x) \in U_n \text{ for infinitely many } n\}.$$

Previous research has mainly concentrated on conformal dynamics. We study a shrinking target problem on a typical self-affine set.

This work is joint with Felipe Ramírez (University of York).

Dynamics of the scenery flow and geometry of measures

Antti Käenmäki
(University of Jyväskylä, Finland)

We present applications of the recently developed ergodic theoretic machinery on scenery flows to classical geometric measure theoretic problems in Euclidean spaces. We shall also quickly review the enhancements to the theory required in our work. Our main results include a sharp version of the conical density theorem, which we reduce to a question on rectifiability. Moreover, we show that dimension theory of measure theoretical lower porosity can be reduced back to the set theoretical analogue. Similar results also hold for other notions of porosities.

Fractal Zeta Functions and Complex Dimensions: A General Higher-Dimensional Theory.

Michel L. Lapidus
(University of California, United States)

We will give some sample results from the new higher-dimensional theory of complex fractal dimensions developed jointly with Goran Radunovic and Darko Zubrinic in the forthcoming research monograph (joint with these same co-authors), with the same title as this talk. We will also explain its connections with the earlier one-dimensional theory of complex dimensions developed, in particular, in the research monograph (by M.

L. Lapidus and M. van Frankenhuijsen) entitled "Fractal Geometry, Complex Dimensions and Zeta Functions: Geometry and Spectra of Fractal Strings" (Springer Research Monographs, Springer, New York, 2013; 2nd rev. and enl. edn. of the 2006 edn.).

In particular, to an arbitrary compact subset A of the N-dimensional Euclidean space (or, more generally, to any relative fractal drum), we will associate new distance and tube zeta functions, as well as discuss their basic properties, including their holomorphic and meromorphic extensions, and the nature and distribution of their poles (or 'complex dimensions'). We will also show that the abscissa of convergence of each of these fractal zeta functions coincides with the upper box (or Minkowski) dimension of the underlying compact set A, and that the associated residues are intimately related to the (possibly suitably averaged) Minkowski content of A. Finally, if time permits, we will discuss and extend to any dimension the general definition of fractality proposed by the authors (and M-vF) in their earlier work, as the presence of nonreal complex dimensions. We will also provide examples of "hyperfractals", for which the 'critical line' Re(s)=D, where D is the Minkowski dimension, is not only a natural boundary for the associated fractal zeta functions, but also consist entirely of singularities of those zeta functions. We may close with a brief discussion of a few of the many open problems stated at the end of the aforementioned forthcoming book.

Spectral property of self-similar sets and measures

Ka-Sing Lau
(Chinese University of Hong Kong, China)

Let μ be a probability measure with compact support K on $\mathbb{R}^d$, μ is called a *spectral measure* if $L^2(\mu)$ admits an orthonormal basis of the form $\{e^{2\pi i\langle\lambda,\cdot\rangle}\}_{\lambda\in\Lambda}$, and if μ is the Lebesgue measure, then K is call a *spectral set*. In the 70's, Fuglede made a conjecture that K is a spectral set iff it is a translational tile. Although the problem was disproved eventually, it generates a lot of interesting questions. In this talk, we report on some development of the problem on the self-similar sets/measures.

Random iteration and disjunctive processes

Krzysztof Leśniak
(Nicolaus Copernicus University in Toruń, Poland)

Let $F = \{f_1, \ldots, f_N : X \to X\}$ be a family of nonexpansive (1-Lipschitz) functions on the complete metric space X. Let $C \subset X$ be a nonempty closed bounded set which is minimal invariant for F, i.e., $F(C) := \bigcup_{i=1}^N f_i(C) = C$, and $F(C_1) = C_1 \Rightarrow C_1 \supset C$ for any closed nonempty $C_1 \subset C$.

Given $x_0 \in X$ and the sequence of symbols (driver) $i_1, i_2, \ldots \in \{1, \ldots, N\}$ we define

- an orbit $x_n := f_{i_n}(x_{n-1})$, $n \geq 1$, and the corresponding
- omega-limit set $\omega((x_n)) := \bigcap_m \overline{\{x_n : n \geq m\}}$.

Recall that a driver $(i_n)_{n=1}^{\infty} \in \{1, \ldots, N\}^{\infty}$ is disjunctive provided it contains all finite words over the alphabet $\{1, \ldots, N\}$.

Theorem. Suppose that the system F:

(B) admits a nonempty closed bounded minimal invariant set, and

(CO) is contractive on orbits, i.e., there exists $(u_1, \ldots, u_k) \in \{1, \ldots, N\}^k$ s.t. $f_{u_k} \circ \ldots \circ f_{u_1}$ is a Lipschitz contraction when restricted to the branching tree

$$\{x_0\} \cup \{f_{i_m} \circ \ldots \circ f_{i_1}(x_0) : i_1, \ldots, i_m \in \{1, \ldots, N\}, m \geq 1\}.$$

If the driver $(i_n)_{n=1}^{\infty}$ of the orbit $(x_n)_{n=1}^{\infty}$ is disjunctive, then $\omega((x_n))$ is a closed minimal invariant set for F.

A discrete time stochastic process $(Z_n)_{n \geq 1}$ is said to be disjunctive provided it generates a disjunctive sequence a.s. The class of such processes includes for example nonhomogeneous Bernoulli schemes and Markov chains (under specific decay conditions), but excludes sequences of identicallly distributed pairwise independent random variables.

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On the Hausdorff dimension faithfulness and the Cantor series expansion

Mykola Lebid

(Bielefeld University, Germany)

We study families Φ of coverings which are faithful for the Hausdorff dimension calculation on a given set E (i. e., special relatively narrow families of coverings leading to the classical Hausdorff dimension of an arbitrary subset of E) and which are natural generalizations of comparable net-coverings. They are shown to be very useful for the determination or estimation of the Hausdorff dimension of sets and probability measures. Motivated by applications in the multifractal analysis of infinite Bernoulli convolutions, we study in details the Cantor series expansion and prove necessary and sufficient conditions for the corresponding net-coverings to be faithful. To the best of our knowledge this is the first known sharp condition of the faithfulness for a class of covering families containing both faithful and non-faithful ones.

Applying our results, we characterize fine fractal properties of probability measures with independent digits of the Cantor series expansion and show that a class of faithful net-coverings essentially wider that the class of comparable ones. We construct, in particular, rather simple examples of faithful families $\mathcal{A}$ of net-coverings which are "non-comparable" to the Hausdorff measure.

Dirichlet heat kernel estimates for diffusions on inner uniform domains

Janna Lierl
(University of Bonn, Germany)

This talk is about two-sided bounds on the Dirichlet heat kernel for diffusions on unbounded inner uniform domains. P. Gyrya and L. Saloff-Coste gave a general approach to this question in the context of (non-fractal) strongly local regular Dirichlet spaces that satisfy volume doubling and Poincaré inequality. They assumed that the metric of the underlying metric measure space is induced by the Dirichlet form, which is not the case for fractal spaces. In this talk I will present extensions of their result to spaces of fractal type. That is, we assume the underlying Dirichlet space satisfies, besides volume doubling, a Poincare inequality PI(Ψ) for a certain time-space scaling function Ψ of polynomial growth, as well as a cut-off Sobolev inequality. This is joint work with Naotaka Kajino.

Sets of large dimension not containing polynomial configurations

András Máthé
(University of Warwick, United Kingdom)

Given countably many multivariate polynomials with rational coefficients and maximum degree d, it is possible to construct a compact set $E \subset \mathbb{R}^n$ of Hausdorff dimension n/d which does not contain finite point configurations corresponding to the zero sets of the given polynomials.

This has interesting applications to distance sets and "angle sets". (Given a set $E \subset \mathbb{R}^n$, consider the set of angles determined by any three points of E.)

A consequence of the above result is that there exists a compact set in $\mathbb{R}^n$ of Hausdorff dimension $n/2$ which does not contain the angle $\pi/2$. (This is known to be sharp if n is even.) There is also a compact set $E \subset \mathbb{R}^n$ of Hausdorff dimension $n/6$ for which the set of angles determined by E is Lebesgue null.

In the other direction, every set of sufficiently large dimension contains an angle ε close to any given angle.

Another corollary is the existence of a compact set $E \subset \mathbb{R}^n$ $(n \geq 2)$ of Hausdorff dimension $n/2$ which does not contain rational distances nor collinear points, for which the distance set is Lebesgue null, moreover, every distance and direction is realised only at most once by E.

Morphometry of Fractals: Scaling of Minkowski Tensors

Klaus Mecke
(University of Erlangen-Nuremberg, Germany)

The 'shape' of fractals is a remarkably incoherent concept and cannot be captured by volume measures alone. A novel approach to morphology and anisotropy of complex spatial structure uses the complete family of tensor-valued Minkowski functionals. Minkowski tensors are generalizations of the well-known scalar intrinsic volumes and share with them the property of being additive. They are explicitly sensitive to anisotropic aspects of shape and are relevant for many physical properties, e.g. for elastic moduli or permeabilities of porous materials. We provide explicit linear-time algorithms to compute these measures for two- and three-dimensional shapes given by triangulations of their bounding surface. Eigenvalue ratios of Minkowski tensors provide a robust and versatile definition of intrinsic anisotropy. Minkowski tensors are particularly suited for the analysis of fractals and can be applied to experimental data of foams, gels or biopolymer networks. The scaling behavior of these Minkowski functionals can be derived for self-similar structures by standard techniques of integral geometry yielding a family of scaling dimensions. Thus, the morphometry of fractals such as random percolation clusters can additionally be characterized by a complete set of scaling amplitudes generalizing naturally the scaling of the 'content' of a spatial structure.

Thermodynamic formalism on a class of fractals with overlaps

Eugen Mihailescu
(Institute of Mathematics of the Romanian Academy, Romania)

In this talk I will give several recent results on the thermodynamic formalism for a class of fractals with overlaps having some form of hyperbolicity. We study the effect of the overlaps on the dimensions of various sections through these fractals, and also on the equilibrium measures supported on them. In general, the number of foldings of these fractals is variable, and we will explain the effect of this behavior on the associated dynamics and geometry.

Multifractal zeta functions

Vuksan Mijovic
(University of St Andrews, United Kingdom)

Talk will be based on joint paper with Lars Olsen in which we introduce multifractal zeta-functions providing precise information of a very general class of multifractal spectra, including, for example, the multifractal spectra of self-conformal measures and the multifractal spectra of ergodic Birkhoff averages of continuous functions. More precisely, we prove that these and more general multifractal spectra equal the abscissae of convergence of the associated zetafunctions.

Two problems on self-similar geometry

Manuel Morán
(Complutense University of Madrid, Spain)

1º) We study under which conditions a self-similar set admits a nontrivial group of isometries. We present a computer program able to generate dynamic images of 3-D self-similar sets with the groups of isometries of the five platonic bodies. 2º) (Joint work with Marta Llorente) We present a code for the computation of the exact packing measure of totally disjointed self-similar sets. We prove the convergence of the algorithm to its target quantity and give some examples of its performance.

A weak local irregularity property in the S^ν spaces

Samuel Nicolay
(University of Liège, Belgium)

Although it has been shown that, from the prevalence point of view, the elements of the S^ν spaces are almost surely multifractal, we show here that they also almost surely satisfy a weak irregularity property: there exists a regularity exponent which is constant for almost every element of S^ν. This is a joint work with Marianne Clausel.

On fractal phenomena connected with infinite linear IFS and properties of non-normal numbers

Roman Nikiforov
(National Pedagogical Dragomanov University, Ukraine)

We establish several new probabilistic, fractal and number theoretical phenomena connected with the Q_∞-expansion which is generated by iterated function systems (IFS) consisting of infinite similitudes with positive ratios q_i such that $\sum_{i=1}^{\infty} q_i = 1$. First of all we show that the system of cylinders of this expansion is, generally speaking, not faithful, i.e., to determine the Hausdorff dimension of a set from the unit interval one is not restricted to consider only coverings consisting of the above mentioned cylinders. We prove sufficient conditions for the non-faithfulness of the family of Q_∞-cylinders. On the other hand, sufficient conditions for the faithfulness of such covering systems are also found.

Based on results related to fine fractal properties of probability measures with independent Q_∞-digits, we study the set $D(Q_\infty)$ of Q_∞-non-normal numbers, i.e., real numbers x such that the asymptotic frequency $\nu_i(x)$ of the digit i in the Q_∞-expansion of x does not exist for at least one Q_∞-digit $i \in \{0, 1, 2, 3, ...\}$. We have shown in particular that this set is of the second Baire category and it is of full Hausdorff dimension.

This is a joint work with Sergio Albeverio, Yuri Kondratiev and Grygoriy Torbin.

Thin and Fat sets in metric spaces

Tuomo Ojala
(University of Jyväskylä, Finland)

Let X be a metric space and $E \subset X$. We call a set E *fat* if it has positive measure for all doubling measures and *thin* if it has zero measure for all doubling measures of the space X. For non-selfsimilar Cantor sets $C(\alpha_n) \subset \mathbb{R}$ defined with sequence (α_n) there is a complete characterization of thinness/fattness in terms of the defining sequence. Now we show that the same characterization holds for very general class of sets in uniformly perfect metric spaces as well. In this talk we give a definition of a quasisymmetrically invariant class of sets that we call (α_n)*-regular* and then characterize thinness and fatness of these sets in terms of the defining sequence (α_n).In addition we give some sufficient conditions that guarantee that at least some of the doubling measures of the ambient space X will remain doubling on the (α_n)-regular set when considered as a metric space of it's own.

Polynomial eigenfunctions of an operator associated to affine IFS

Helena Peña
(University of Greifswald, Germany)

To any IFS on $X = \mathbb{R}^n$ or $\mathbb{C}^n$ with affine transformations, an operator acting on the space of continuous functions on X can be associated by duality. For the case $n = 1$ we can explicitly determine all polynomial eigenfunctions and their eigenvalues. These are related to the invariant measure of the IFS. As an example, we consider the operator of this kind associated to Bernoulli convolutions and show how it yields polynomial approximations of the convolution measure in case of absolute continuity.

Fractal curvatures: tube formulas and measurability

Erin Pearse
(United States, United States)

I will present recent results on Minkowski measurability of fractals, as obtained via tube formulas, and the lattice/nonlattice dichotomy for fractal sprays and tilings.

Uniqueness of Eigenforms on Fractals

Roberto Peirone
(University of Rome Tor Vergata, Italy)

In this talk, I will describe some recent work of mine about the uniqueness of eigenforms (with respect to the renormalization operator with weights Λ_r) on finitely ramified fractals. Note the eigenforms correspond to the self-similar energies on all of the fractal. Therefore, the results provide corresponding results on uniqueness of self-similar energies. I will describe an explicit necessary and sufficient condition for the uniqueness of a given eigenform E in terms of E. Moreover, a geometric sufficient condition for the nonuniqueness of the eigenform for any set of weights r will follow. We can deduce from it many examples of fractals where the eigenform exists but is not unique, that are not trees (and with a structure sometimes very different from a tree). Finally, we can deduce examples of fractals where the uniqueness of the eigenform depends on the weights.

Rescaling invariants of dynamical systems

Yakov Pesin
(Pennsylvania State University, United States)

I will discuss a general approach to rescaling some invariants of dynamical systems such as topological and metric entropies, Lyapunov exponents, etc. It allows asymptotic rates of the general form $e^{\alpha a(n)}$ determined by an arbitrary monotonically increasing "scaling" sequence $a(n)$ and covers the standard case of exponential scale corresponding to $a(n) = n$ as well as the cases of zero and infinite entropy. This is a joint work with Yun Zhao.

Integrated density of states for Poisson perturbations of Markov processes on the Sierpiński gasket

Katarzyna Pietruska-Paluba
(University of Warsaw, Poland)

The integrated density of states is an object central to the investigation of random Hamiltonians in infinite volume. It is the limit of the counting measures based on the spectra of finite-volume random Hamiltonians, normalized with respect to the volume. It has been thoroughly investigated in the classical setting (e.g. when the Hamiltonian is the Laplacian – corresponding to the Brownian motion). At present, we focus our attention on the subordinate Brownian motions on the infinite Sierpiński gasket, evolving in the environment influenced by a killing Poissonian potential. Such a potential will be given by

$$V^{\omega}(x, y) = \int_{\mathcal{G}} W(x, y) \mathrm{d}\mu^{\omega}(y),$$

where $\mathcal{G}$ denotes the gasket, μ^{ω} is the random Poisson point measure on $\mathcal{G}$, and W is the profile function.

We prove that for a vast class of subordinate Brownian motions and potentials the integrated density of states exists, is nonrandom, and is the same for both the

Dirichlet and the Neumann boundary conditions. We provide some concrete examples of potentials, including potentials with infinite-range profiles.

Our methods are also suitable for investigating the killing Poissonian obstacles instead of a potential interaction. We obtain similar results in that case too.

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The Ihara zeta function for infinite graphs II

Felix Pogorzelski

(University of Jena, Germany)

We give a general definition of the Ihara zeta function on general measurable graphs. Our approach unifies previous definitions on finite and infinite graphs. With this notion at hand we are able to represent these zeta functions as determinants of certain geometric operators. Furthermore, one can establish the approximation along Benjamini-Schramm convergent sequences. The first talk is devoted to describing the geometric setting and to defining the zeta function, while the second one deals with approximation results.

Scaling exponents of curvature measures

Dusan Pokorny

(Charles University in Prague, Czech Republic)

Fractal curvatures of a subset F of $\mathbb{R}^d$ are roughly defined as suitably rescaled limits of the total curvatures of its parallel sets F_ε as ε tends to 0 and have been studied in the last years in particular for self-similar and self-conformal sets. This previous work was focussed on establishing the existence of (averaged) fractal curvatures and related fractal curvature measures in the generic case when the kth curvature measure $C_k(F_\varepsilon, .)$ scales like ε^{k-D}, where D is the Minkowski dimension of F. In the talk we will discuss the non-generic situation when the scaling exponents do not coincide with the dimension. We will demonstrate that the possibilities for non-generic behaviour are rather limited and introduce the notion of local flatness, which allows a geometric characterization of non-genericity in $\mathbb{R}$ and $\mathbb{R}^2$. We expect local flatness to be characteristic also in higher dimensions. The results also enlighten the geometric meaning of the scaling exponents. The results are a joint work with Steffen Winter.

The Apollonian Gasket and its dimension

Mark Pollicott
(Warwick University, United Kingdom)

The Apollonian Gasket is a classical example of a non-linear fractal. Starting from three mutually tangent circles inside and tangent to the unit circle, there is a familiar iterative construction given by repeated adding more circles into the gaps between existing circles. The limiting fractal has dimension d = 1.3057 ... In this talk we will describe the significance of the dimension in terms of the asymptotics of the radii of the infinitely many circles. We will also describe other statistical properties of these radii.

Entropy of Lyapunov-optimizing measures of some matrix cocycles

Michal Rams
(IMPAN, Poland)

Let $\{A_i\}_{i=1}^k$ be a finite family of matrices from $GL(2,\mathbb{R})$. For a sequence $\omega = \omega_1\omega_2 \ldots \in \{1,\ldots,k\}^{\mathbb{N}}$ consider the maximal Lyapunov exponent of the corresponding product of matrices:

$$\lambda(\omega) = \lim_{n\to\infty} \frac{1}{n} \log ||A_{\omega_n} \ldots A_{\omega_1}||$$

(wherever it is defined). We want to investigate the sequences for which the Lyapunov exponent takes an extremal (minimal or maximal) value.

This is a natural generalization of standard multifractal questions, with the important difference that the product of matrices is not commutative (hence, the Lyapunov exponent behaves very differently than a usual Birkhoff limit for a real-valued potential).

The result: under some natural assumptions we are able to show that the sets of sequences for which the extremal values of the Lyapunov exponent are achieved are small, in the following sense: all the exponent-minimizing and exponent-maximizing shift-invariant measures have entropy zero. This is a joint work with Jairo Bochi.

A Variational Principle for Measurable Potentials

Marc Rauch
(University of Jena, Germany)

We introduce the notion of topological pressure for sequences of measurable functions on compact metric spaces with discrete dynamics. For this notion there exists under fairly general assumptions a corresponding variational principle, which includes and extends the variational principles for the classical case as well as (asymptotically) subadditive and superadditive sequences. We also provide an inverse variational principle and a bound for the pressure on α-level sets.

Box-counting zeta functions and complex dimensions

John Rock
(United States, United States)

Key results of the theory of complex dimensions of ordinary fractal strings developed by Lapidus and van Frankenhuijsen illuminate deep connections between geometric and spectral oscillations of such strings and the structure of their complex dimensions. The box-counting zeta functions which will be discussed in this talk provide a context that allows for investigation of connections between geometric oscillations of bounded sets in Euclidean space and their complex dimensions. Topics to be discussed in some detail include the recovery of upper box-counting dimension as an abscissa of convergence, a closed form for the zeta functions of self-similar sets that satisfy a pairwise disjoint condition, and connections between complex dimensions and Minkowski measurability in these settings.

Measure and Hausdorff dimension of randomized Weierstrass-type functions

Julia Romanowska
(University of Warsaw, Poland)

In my talk I will consider functions of the type

$$f(x) = \sum_{n=0}^{\infty} a_n g(b_n x + \theta_n),$$

where (a_n) are independent random variables uniformly distributed on $(-a^n, a^n)$ for some $0 < a < 1$, $b_{n+1}/b_n \geq b > 1$, $a^2 b > 1$ and g is a C^1 periodic real function with finite number of critical points in every bounded interval. I will prove that the occupation measure for f has L^2 density almost surely. Furthermore, the Hausdorff dimension of the graph of f is almost surely equal to $D = 2 + \log a / \log b$ provided $b = \lim_{n\to\infty} b_{n+1}/b_n > 1$ and $ab > 1$.

Basin of attraction of general IFSs

Miroslav Rypka
(Palacky University in Olomouc, Czech Republic)

We will discuss basins of attraction of IFSs consisting of continuous mappings in metric spaces. Basin of attraction in the case of contractions in a complete metric space is the whole space. In general, the situation is not so simple, even the definition of attractor and basin of attraction is worth attention.

Equidistribution modulo 1 and Fourier transforms of equilibrium states

Tuomas Sahlsten
(The Hebrew University of Jerusalem, Israel)

We prove that any Gibbs equilibrium state for the Gauss map with a thin tail and Hausdorff dimension at least 1/2 has Fourier transform decaying to 0 with a polynomial rate. The proof is based on investigating an example of Kaufman from the 1980s where he constructed a special measure on the set of badly approximable numbers and deducing large deviation bounds for the countable Markov shift coming from the thin tail assumption. Thanks to Davenport-Erdös-LeVeque criterion on equidistribution any such a measure μ has the sequence $(s_k x)_{k \in N}$ equidistributing modulo 1 at μ a.e. x for any strictly increasing sequence of integers s_k. Especially in the case $s_k = n^k$ and when mu is supported on a finite alphabet set, we recover a recent n-normality result for Gibbs measures by Hochman-Shmerkin in the case of Gauss map obtained using scenery flow techniques. We also discuss about the obstacles in the method that seem to prevent us from extending the result beyond the Gauss map.

Based on joint work with Thomas Jordan (Bristol)

Intermediate β-shifts of finite type

Anthony Samuel
(University of Bremen, Germany)

An elementary classification, in terms of the of the kneading invariants, of the linear transformations $T_{\beta,\alpha} : x \mapsto \beta x + \alpha \bmod 1$ for which the corresponding intermediate β-shift is of finite type will be presented. (Here we assume that $1 < \beta < 2$ and $0 \leq \alpha \leq 2 - \beta$.) We will then show how this characterisation can be employed to construct a class of pairs (β, α) such that the intermediate β-shift is of finite type but for the map $T_{\beta,\alpha}$ is not transitive, which is in contrast with the situation of the corresponding greedy and lazy β-transformations.

[This is join work with Bing Li (South China University of Technology) and Tuomas Sahlsten (The Hebrew University of Jerusalem).]

Calculus, transport and Heat flow in metric measure spaces

Giuseppe Savare
(University of Pavia, Italy)

We will present some applications of Optimal Transport and the Hopf-Lax semigroup to the study of the Sobolev space $W^{1,2}(X, d, m)$, the related Cheeger energy and the Heat flow in a general metric measure space (X, d, m). The particular case of spaces with Riemannian Ricci curvature bounded from below will also be discussed. (In collaboration with Luigi Ambrosio and Nicola Gigli)

The Ihara zeta function for infinite graphs I

Marcel Schmidt
(University of Jena, Germany)

We give a general definition of the Ihara zeta function on general measurable graphs. Our approach unifies previous definitions on finite and infinite graphs. With this notion at hand we are able to represent these zeta functions as determinants of certain geometric operators. Furthermore, one can establish the approximation along Benjamini-Schramm convergent sequences. The first talk is devoted to describing the geometric setting and to defining the zeta function, while the second one deals with approximation results.

Prescription of multifractal spectrum

Stéphane Seuret
(Paris 12 Val de Marne University, France)

In this talk, I will explain how to construct measures with prescribed multifractal spectrum. The spectra that we are able to reach are suprema of a countable number of step function, and thus it includes all continuous functions. We discover a surprising constraint on the multifractal spectrum of homogeneous measures: the range of exponents less than one must be an interval. The proof is based on maximal inequalities and geometric measure arguments. This is a joint work with Z. Buczolich.

Absolute continuity of self-similar measures

Pablo Shmerkin
(Torcuato Di Tella University, Argentina)

A self-similar measure on $\mathbb{R}^d$ has associated a number known as its similarity dimension. When this number is $< d$, then the measure is well-known to be singular. The problem I will address in this talk is: what can we say about absolute continuity when the similarity dimension is $> d$? This problem goes back to Erdös' pioneering study of Bernoulli convolutions in 1939-1940.

Even today we do not know how to determine if a given self-similar measure is absolutely continuous or not (outside of very special cases) so a long and fruitful line of research has focused on giving bounds on the size of the set of exceptional parameters for which the self-similar measure is singular (in the regime where similarity dimension $> d$).

I will present a number of results to the effect that for several natural classes of self-similar measures this exceptional set is much smaller than previously known; for Bernoulli convolutions and other natural classes, it is even zero-dimensional. The proofs combine evolutions of Erdös' original idea with cutting-edge recent results of Mike Hochman.

Part of the talk is based on joint work with B. Solomyak.

Fractals and Complex Networks

Karoly Simon
(Budapest University of Technology and Economics, Hungary)

In this talk, I mention some ideas and methods of Fractal Geometry which have been used to study the structure of complex networks.

The argument of Hausdorff-dimension for the representation of the strategies playing in the stochastic prisoner's dilemma on probabilistic graphs with the Fruchterman-Reingold algorithm

Levente Simon
(Babecs-Bolyai University, Romania)

The presentation focuses on the simulation analysis of the stochastic prisoner's dilemma with two players. The two states of a game are: the initial is the interpreted by Axelrod and the second is the multiplied case with a natural non-zero number, called punishment multiplier. Hypothesis: if both cooperate currently, the next state in the sequence of games will be the multiplied; if both defects, the next will be the initial state; otherwise, the states has the same probability in the next step. The target is to find the minimal value of the multiplier in the case of the mean of the payoffs characterizes the sequences and this causes the win of the cooperative or teeth-for-teeth strategies against the defectives, independently from the distribution of the strategies. The strategies on the Erdős-Rényi, Watts-Strogatz and Barabási-Albert graphs are represented with a deterministic, a probabilistic and the iterative algorithm of Fruchterman and Reinghold. The approximated Hausdorff-dimension of these representations arguments for the usage of the Fructerman-Reingold algorithm in the case of the strategies of the stochastic prisoner's dilemma on Barabási-Albert graphs.

About the multifractal nature of Cantor's bijection

Laurent Simons
(University of Liège, Belgium)

In this talk, we present the Cantor's bijection between the irrational numbers of the unit interval $[0, 1]$ and the irrational numbers of the unit square $[0, 1]^2$. We explore the regularity and the fractal nature of this map. This talk is based on a joint work with S. Nicolay.

Space-filling curves, expanding maps on the circle and geodesic laminations.

Victor Sirvent
(Simón Bolívar University, Venezuela)

In this talk we consider a class of connected fractals that admit a space filling curve. We prove that these curves are Hölder continuous and measure preserving. To these space filling curves we associate geodesic laminations satisfying among other properties that points joined by geodesics have the same image in the fractal under the space filling curve. The laminations help us to understand the geometry of the curves. The construction of the laminations is associated to a family expanding dynamical system on the circle. This family allows us to define expanding dynamical systems on the laminations. We explore the relations between the geometric properties of the laminations, the space-filling curves and the dynamical properties of the expanding maps.

On sharp condition for packing dimension preservation under transformations generated by random s-adic expansion

Oleksandr Slutskyi
(TNPU, Ukraine)

My talk is devoted to the development of theory of packing dimension preserving (PDP) transformations. It is proven that any bi-Lipshitz transformation is PDP. A class of distribution functions of random variables with independent s-adic digits is studied in details. Necessary and sufficient conditions for packing dimension preservation are proven.

Absolutely Continuous Convolutions of Singular Measures and an Application to the Square Fibonacci Hamiltonian

Boris Solomyak
(University of Washington, United States)

We prove for the square Fibonacci Hamiltonian that the density of states measure is absolutely continuous for almost all pairs of small coupling constants. This is obtained from a new result we establish about the absolute continuity of convolutions of measures arising in hyperbolic dynamics with exact-dimensional measures. This is a joint work with David Damanik and Anton Gorodetski.

Stochastic spline fractal interpolation function

Anna Soos
(Babecs-Bolyai University, Romania)

We construct a probabilistic spline fractal interpolation function f using values of $f^{(k)}$, $k = 1, 2, ...l$ at the endpoints of the interval $[x_0, x_n]$. With this type of interpolation function one can use the fractal dimension of f in the analysis of experimental data.

Diffusion on non-self-similar Sierpinski carpets

Benjamin Steinhurst
(McDaniel College, United States)

The existence of diffusions on self-similar Sierpinski carpets is well known. In recent years estimates for the associated heat kernels have been a major topic of study. MacKay, Tyson, and Wildrick recently have shown using Lipschitz analysis the existence of a Gaussian diffusion on a class of non-self-similar carpets. However there is a large gap between these non-self-similar carpets and the statistically self-similar carpets that Hambly, Kumagai, Kusuoka, and Zhou have studied. In this talk I will address diffusion on these intermediate carpets showing their existence and immediate properties.

On measures driven by Markov chains

Andrzej Stos
(Blaise Pascal University, France)

We study measures on $[0, 1]$ which are driven by a finite Markov chain and which generalize the famous Bernoulli products. We propose a hands-on approach to determine the structure function τ and to prove that the multifractal formalism is satisfied. Formulas for the dimension of the measures and for the Hausdorff dimension of their supports are also provided.

Intersection properties of spatially independent martingale measures

Ville Suomala
(University of Oulu, Finland)

We consider a general class of fractal sets and measures obtained as limits of certain spatially independent martingale measures. Under additional geometric assumptions, these random fractals fulfill the statements of Marstrand-Mattila type projection theorems without any exceptional directions. Among other things, the orthogonal projections of these measures are shown to be uniformly Hölder continuous. Further, we find the sharp dimension threshold (in terms of dimension) for these random sets to containing given patterns (all progressions, all angles etc.) There are also many other applications; For instance, we prove the existence of random sets $A \subset \mathbb{R}^d$ of any dimension $0 < s < d$ such that the dimension of $E \cap A$ is at most $\dim A + \dim E - d$ for any self-similar set E satisfying the open set condition. This is joint work with Pablo Shmerkin.

The first step of the Brownian sheep

András Telcs
(University of Pannonia, Hungary)

In our talk we discuss isoperimetric inequalities for weighted graphs. In particular we present characterization of sets with minimal capacity and expected lifetime of the random walks.

Some rigidity properties of self-simiar Jordan arcs

Andrey Tetenov
(Gorno-Altaisk State University, Russia)

As it was shown by the author, each self- similar Jordan arc can be represented as an attractor of some self-similar multizipper.

Now we prove that if a multizipper has a strongly connected structural graph and has similarity dimension 1, then its attractor consists of straight line segments.

Basing on this result we prove that each self-similar Jordan arc γ for which a set of points at which there is a hyperplane, weakly transversal to γ, is dense in γ, is a straight line segment.

Therefore Jordan arcs γ in $\mathbb{R}^d$, which are different from a line segment cannot be projected to a line bijectively.

On the Hausdorff dimension faithfulness of net coverings and fractal properties of essentially non-normal numbers

Grygoriy Torbin
(National Pedagogical Dragomanov University, Ukraine)

It is well known that in many situations the determination (or even estimations) of the Hausdorff dimension for sets from a given family or even for a given set is a rather non-trivial problem. To simplify the calculation of the this dimension it is extremely useful to have an appropriate and a relatively narrow family of admissible coverings which lead to the same value of the dimension.

Let Φ be a *fine family of coverings* on $[0, 1]$, i.e., a family of subsets of $[0, 1]$ such that for any $\varepsilon > 0$ there exists an at most countable ε - covering $\{E_j\}$ of $[0, 1]$ with $E_j \in \Phi$. Let us recall that *the α-dimensional Hausdorff measure* of a set $E \subset [0, 1]$ w.r.t. a given fine family of coverings Φ is defined by

$$H^\alpha(E, \Phi) = \lim_{\varepsilon\to 0}\left[\inf_{|E_j|\le\varepsilon}\left\{\sum_j |E_j|^\alpha\right\}\right] = \lim_{\varepsilon\to 0} H^\alpha_\varepsilon(E,\, \Phi),$$

where the infimum is taken over all at most countable ε-coverings $\{E_j\}$ of E, $E_j \in \Phi$.

The nonnegative number

$$\dim_H(E,\, \Phi) = \inf\{\alpha : \; H^\alpha(E,\, \Phi) = 0\}$$

is called the Hausdorff dimension of the set $E \subset [0, 1]$ w.r.t. a family Φ.

Definition. A fine covering family Φ is said to be *faithful family of coverings (non-faithful family of coverings)* for the Hausdorff dimension calculation on $[0,1]$ if

$$\dim_H(E,\Phi) = \dim_H(E), \quad \forall E \subseteq [0,1]$$

$$(\text{resp. } \exists E \subseteq [0,1] : \dim_H(E,\Phi) \neq \dim_H(E)).$$

It is clear that any family Φ of comparable net-coverings (i.e., net-coverings which generate comparable net-measures) is faithful. Conditions for a fine covering family to be faithful were studied by many authors (see, e.g., papers by A. Besicovitch, P. Billingsley, C. Cutler, M.Pratsiovytyi, S. Albeverio, G. Torbin, R.Nikiforov, M. Lebid, M. Ibragim). The family of cylinders of the classical continued fraction expansion can probably be considered as the first (and rather unexpected) example of non-faithful one-dimensional net-family of coverings ([1]). By using approach, which has been invented by Yuval Peres to prove non-faithfulness of the family of continued fraction cylinders ([1]), in [2] authors have proven the non-faithfulness for the family of cylinders of Q_∞-expansion with polynomially decreasing elements $\{q_i\}$. Necessary and sufficient conditions for the family of cylinders generate by the Cantor series expansion were found in [4] (to the best of our knowledge this gives the first necessary and sufficient condition of the faithfulness for a class of covering families containing both faithful and non-faithful ones). During the talk we present some new general necessary and sufficient conditions for a covering family to be faithful and new techniques for proving faithfulness/non-faithfulness for the family of cylinders generated by different expansions of real numbers. Connections between faithfulness of net coverings and the theory of DP-transformations (i.e., transformations preserving the Hausdorff dimension) will also be discussed.

The second direction of the talk will be devoted to the study of dependence of topological and fractal properties of sets of essentially non-normal numbers (i.e. those real numbers for which the asymptotic frequencies of all digits do not exist) on the chosen expansions of real numbers. It is known (see, e.g., [3] and references therein) that for many expansions such set is of full Hausdorff dimension and is of the second Baire category. We present some natural expansions of real numbers for which the set of essentially non-normal numbers is of zero Hausdorff dimension ([5]) and which generate non-faithful net-coverings.

The talk is based on joint research with S. Albeverio, I.Garko, M.Ibragim, Yu. Kondratiev, M.Lebid, R. Nikiforov, Yu.Peres.

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Differentiability of self-conformal devil's staircases

Sascha Troscheit
(University of St Andrews, United Kingdom)

In this talk we will present results about the probability distribution function of a Gibbs measure supported on a self-conformal set given by an iterated function system applied to a compact subset of $\mathbb{R}$ (devil's staircase). We use thermodynamic multifractal formalism to calculate the Hausdorff dimension of the sets S_0^α, S_∞^α and S^α, the set of points at which this function has, respectively, Hölder derivative 0, ∞ or no derivative in the general sense. This extends recent work by considering Gibbs measures generated by potentials independent of the geometric potential.

Convex/Concave Bivariate Rational Fractal Interpolation Function

Nallapu Vijender
(Indian Institute of Technology Madras, India)

Fractal interpolation is more general than any classical piecewise interpolation due to the presence of the scaling factors that describe smooth or non-smooth shape of the fractal curve/surface. We develop a bivariate rational fractal interpolation function by using the blending functions and rational cubic fractal interpolation functions (FIFs) with 2-shape parameters in each sub-interval along the grid lines of rectangular domain $\mathcal{D}$. The properties of blending functions and $\mathcal{C}^1$-smoothness of rational cubic FIFs render the $\mathcal{C}^1$-smoothness to our bivariate rational FIF. The convergence of proposed bivariate rational FIF towards an original function in $\mathcal{C}^2(\mathcal{D})$ is studied by calculating an upper bound for the error of the interpolation. The scaling factors and shape parameters seeded in the rational cubic FIFs are constrained so that these rational cubic FIFs are convex/concave whenever the univariate data sets along the grid lines are convex/concave. For these constrained scaling factors and shape parameters, our bivariate rational FIF is convex/concave to given convex/concave surface data. Finally, our theoretical results are implemented for the construction of the convex fractal surfaces over a rectangular domain.

** This is joint work with Dr. A. K. B. Chand and Prof. M. A. Navascues

A New Approach to Positivity Preserving Interpolation Using Smooth α-Fractal Functions

Puthan Veedu Viswanathan
(Indian Institute of Technology, India)

Fractal Interpolation Functions (FIFs) produce interpolants that share smoothness or non-smoothness quality of the data generating function. Smooth FIFs (fractal splines) constitute an advance in the techniques of interpolation, since the traditional non-recursive methods of real-data interpolation can be generalized by means of smooth fractal techniques. Due to some reasons, perhaps the physical situation which the interpolant is intended to model, we wish the graph of the interpolant to preserve certain geometric properties such as positivity, monotonicity, convexity, and unimodality of the data. The field of computer aided design has provided the impetus for the development of *shape preserving interpolation*.

In spite of the versatility and flexibility offered by the fractal interpolation schemes, standard procedures for constructing fractal splines lead to functions which do not reproduce shape properties inherent in the data. Owing to the *tension effect*, rational FIFs with shape parameters form suitable vehicle for an easy and elegant exposition of fractal functions to the field of shape preserving interpolation. In this paper, we propose a unified approach for the generalization of the traditional non-recursive shape preserving rational spline interpolation schemes using positivity as an example of shape. Our treatment consists of two steps. In the first step, we develop a method to construct $\mathcal{C}^p$-continuous FIFs using α-fractal technique. In this procedure, we employ a finite sequence of *base functions* in contrast to a single base function method adopted in the literature. The application of a finite family of base functions facilitates to incorporate shape parameters in the fractal spline structure. In the second step, we identify conditions on the scaling parameters so that the fractal function f^α defined through a finite sequence of base functions retain the positivity property of the function f being perturbed. This positivity preserving fractal perturbation can then be applied to obtain fractal analogues of various positivity preserving rational spline schemes available in the literature. However, for the sake of definiteness, we illustrate our procedure by finding a fractal generalization f^α of the rational cubic spline with quadratic denominator f studied in the literature, and finding conditions for the positivity of f^α. For approximating a positive function Φ whose derivative Φ' has irregularity in a dense subset of the interpolation interval, the proposed positive fractal spline outperforms the corresponding classical counterpart.

Note: This is a joint work with A. K. B. Chand.

Regularity results for solutions in the Koch snowflake domain.

Maria Agostina Vivaldi
(Sapienza University of Rome, Italy)

In this talk we consider a sequence pre-fractal domains approximating the Koch snowflake. These pre-fractal domains are polygonal, non convex and with an increasing number of sides. More precisely, the boundary is a polygonal curve developing at the limit a fractal geometry. As consequence, it is the union of an increasing number of graphs.

The (classical) regularity results, in weighted Sobolev spaces, for the solutions of the PDEs provide estimates that involve constants that diverge as the number of graphs become infinity. In this talk we discuss uniform estimates, in weighted Sobolev spaces, for the solutions of the PDEs on the pre-fractal domains as well as regularity results (in weighted Sobolev spaces) for the solutions of the PDEs on the snowflake domain. Our proof combines the pioneer results of Kondratiev with the sophisticated techniques introduced for the NTA domains.

The Hausdorff dimension of the range and the graph of an operator semistable Lévy process

Lina Wedrich
(University of Düsseldorf, Germany)

Let $X = (X_t)_{t\geq 0}$ be a Lévy process on $\mathbb{R}^d$. The process X is called operator semistable if it fulfills the following time-scaling property

$$\{c^E X(t)\}_{t\geq 0} \stackrel{\text{fd}}{=} \{X(ct)\}_{t\geq 0} \quad \text{for some } c > 1$$

and some linear operator E on $\mathbb{R}^d$ called the exponent. Here $\stackrel{\text{fd}}{=}$ denotes equality of all finite dimensional marginal distributions. If the equation from above holds for all $c > 0$ the process is called an operator stable Lévy process. For further details on this subject see [2].

For an arbitrary Borel set $B \subseteq \mathbb{R}^d$ Meerschaert and Xiao [3] have already determined the Hausdorff dimension of the range $X(B)$ for the special case of an operator stable Lévy process.

By refining their arguments we determine the Hausdorff dimensions of the range and the graph of an operator semistable Lévy process in terms of the real parts of the eigenvalues of E and the Hausdorff dimension of B.

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Local characterization of Minkowski contents

Steffen Winter
(Karlsruhe Intitute of Technology, Germany)

In some joint work with Jan Rataj we have shown recently that the Minkowski content of a bounded set in R^d (with volume zero) can be characterized completely in terms of the asymptotic behaviour of the surface area of its parallel sets, which was a kind of surprise as a similar characterization was known to hold for upper but to fail for lower Minkowski contents.

In the talk, I will discuss localizations of these characterization results. Using the notion of metrically associated sets, the asymptotic behaviour of the local parallel volume can be understood in terms of some suitably defined local surface area. This allows for instance to characterize relative Minkowski contents in terms of surface area. Moreover, I will present a measure version of this result: Viewing the Minkowski content as a locally determined measure, this measure can be obtained as a weak limit of suitably rescaled surface measures of close parallel sets. Such measure relations had been observed before for self-similar and self-conformal sets. They are now established for arbitrary bounded sets. The proofs are based on a localization of a theorem by Stacho.

Branching dynamical systems and slices through fractals

Rüdiger Zeller
(University of Greifswald, Germany)

Bernoulli convolutions and β-representations are associated to multivalued maps consisting of two linear expanding functions with branching orbits which we call branching dynamical systems. Schmidt proved in 1980 that the greedy β-representation of a given x is eventually periodic iff $x \in \mathbb{Q}(\beta)$, in case that β is a Pisot unit. For the case of the golden mean we show that countably many points possess orbits with growth rates larger than the generic growth rate, which is less than $2/\beta \approx 1.24$ (Feng and Sidorov, 2009). The growth rates generated by the first easiest orbits are $\sqrt{\beta} \approx 1.27$ which is the maximal growth rate, $\sqrt[3]{2} \approx 1.26$ and $\sqrt[7]{5} \approx 1.26$.
Ergodic theorems for intersections of lines with Sierpinski's carpet and triangle were studied by Simon, Manning, Bárány and Ferguson. Finite type intersections of Sierpinski carpet and lines were investigated by Li (1997) and Xi and Wen (2010). In the Pisot-case a generalisation of Schmidt's theorem to branching dynamical systems allows to characterise finite type intersections of hyperplanes with self-similar sets. We discuss the geometry of a number of three-dimensional fractals as well as new constructions appearing as intersections with four-dimensional fractals.