Mathematical Statistics, Winter term 2018/19

Problem sheet 5

12) Let $X_1, \ldots, X_n$ be independent with $X_i \sim \mathrm{Uniform}[\theta, \theta+1]$, where $\theta \in \Theta = \mathbb{R}$. Let π be a prior distribution for θ with an everywhere positive density p.

    Find the Bayesian estimator under squared error loss.

13) Let $X_1, \ldots, X_n$ be i.i.d. with $P_\theta(X_i = 1) = \theta = 1 - P_\theta(X_i = 0)$ and let $\widehat{\theta}_{\alpha,\beta} = \frac{\sum_{i=1}^n X_i + \alpha}{n+\alpha+\beta}$ be the Bayesian estimator under squared error loss and prior distribution $\pi = \mathrm{Beta}(\alpha, \beta)$.)

    (i) Show that $\widehat{\theta}_{\alpha,\beta}$ is admissible for all $\alpha, \beta > 0$.

    (ii) Show that $\widehat{\theta}_{\sqrt{n}/2,\sqrt{n}/2}$ has a constant risk function. Is $\widehat{\theta}_{\sqrt{n}/2,\sqrt{n}/2}$ a minimax estimator in $\Theta = [0, 1]$?

14) Let $X_1, \ldots, X_n$ be i.i.d. with $P_\theta(X_i = 1) = \theta = 1 - P_\theta(X_i = 0)$, where $\theta \in \Theta = (0, 1)$.

    (i) Show that $\bar{X}_n = n^{-1} \sum_{i=1}^n X_i$ is admissible under the loss function L with $L(t, \theta) = (t-\theta)^2/(\theta(1-\theta))$.
    Hint: Show first that $\bar{X}_n$ is the Bayesian estimator under the loss function L and prior distribution $\pi = \mathrm{Uniform}((0, 1))$. Don't forget to show that $\theta \mapsto E_\theta L(T, \theta)$ is continuous for all estimators T.

    (ii) Conclude from (i) that $\bar{X}_n$ is also admissible under the squared error loss.