

Mathematical Statistics, Winter term 2018/19

Problem sheet 6

15) Let  $X_1, \dots, X_n$  be i.i.d. with  $E_\theta X_i = \theta$  and  $E_\theta X_i^2 < \infty \forall \theta \in \Theta$ , where  $\Theta$  is any nonempty subset of  $\mathbb{R}$ .

(i) Under the squared error loss, show that any estimator of the form  $\bar{X}_n + b$  with  $b \neq 0$  being a constant is inadmissible.

(ii) Under the squared error loss, show that any estimator of the form  $a\bar{X}_n + b$  with constants  $a > 1$  and  $b \in \mathbb{R}$  is inadmissible.

16) Let  $X \sim \text{Bin}(\theta, p)$ , where  $\theta \in \Theta = \{0, 1, \dots\}$  and  $p \in (0, 1)$  being fixed.

For the prior distribution  $\pi = \text{Poisson}(\lambda)$ ,  $\lambda > 0$ , compute the posterior distribution  $P^{\theta|X=k}$  and the Bayesian estimator under the squared error loss.

17) Let  $X \sim P_\theta^X = \text{Poisson}(\theta)$ , where  $\theta \in \Theta = \mathbb{R}$ .

(i) Compute the Fisher information of the family  $\{P_\theta^X: \theta \in \Theta\}$ .

(ii) Compute the quadratic risk of the estimator  $T(X) = X$  of the parameter  $\theta$ .

*Hint: Compute first  $E_\theta X$  and  $E_\theta[X(X-1)]$ .*

18) Let  $X \sim P_\theta^X = \mathcal{N}(\theta, \sigma^2)$ , where  $\theta \in \Theta = \mathbb{R}$  and  $\sigma^2 > 0$  being fixed.

Does the family of distributions  $\{P_\theta^X: \theta \in \mathbb{R}\}$  satisfy the conditions (R1) and (R2)? Compute  $I(\theta)$ .