Mathematical Statistics, Winter term 2018/19
Problem sheet 7

19) Let $X_1, \ldots, X_n$ be i.i.d. with $P_\theta(X_i = 1) = \theta = 1 - P_\theta(X_i = 0)$, where $\theta \in \Theta = (0,1)$.

Show with the aid of Theorem 1.3.6 that $\bar{X}_n = n^{-1} \sum_{i=1}^n X_i$ is admissible in the class of all estimators under the squared error loss.

20) Let $X_1, \ldots, X_n$ be i.i.d. with $P_\theta^{X_i} = \mathcal{N}(\theta, \sigma^2)$, where $\sigma^2 > 0$ is fixed and $\theta \in \Theta := (a,b)$, $-\infty \leq a < b \leq \infty$.

For which values of a and b is $\bar{X}_n = n^{-1} \sum_{i=1}^n X_i$ admissible under squared error loss?

21) Let $X_1, \ldots, X_n$ be i.i.d. with $P_\theta^{X_i} = \mathcal{N}(\theta, \sigma^2 I_d)$, where $\theta \in \Theta$ and $\sigma^2 > 0$ being fixed. Let Θ be an open subset of $\mathbb{R}^d$.

(i) Compute the Fisher information of the family $\{P_\theta^{X_i} \colon \theta \in \Theta\}$.

(ii) Show that $T = n^{-1} \sum_{i=1}^n X_i$ is a best unbiased estimator of θ w.r.t. the matrix risk, $E_\theta[(T - \theta)(T - \theta)^T]$.

(iii) Show that T is also the best unbiased estimator under the loss function L with $L(t, \theta) = \|t - \theta\|^2$.