Mathematical Statistics, Winter term 2018/19

Problem sheet 8

22) Let $X_1, \ldots, X_n$ be i.i.d. with $X_i \sim \text{Uniform}([\theta_1, \theta_2])$, where $-\infty < \theta_1 < \theta_2 < \infty$.

   (i) Compute the moment estimator of $\theta = (\theta_1, \theta_2)^T$.

   (ii) Compute the maximum likelihood estimator of θ.

23) Let $(X_i)_{i\in\mathbb{N}}$ be a sequence of independent and identically distributed random variables with $X_i \sim \mathcal{N}(\mu, \sigma^2)$, where $\theta = (\mu, \sigma^2)^T$ und $\Theta = \mathbb{R} \times (0, \infty)$. Let $\widehat{\theta}_{n,ML} = \widehat{\theta}_{n,ML}(X_1, \ldots, X_n)$ be the maximum likelihood estimator of θ.

   (i) Compute the Fisher information matrix $I(\theta)$ of the family $\{P_\theta^{X_1} \colon \theta \in \Theta\}$.

   (ii) Show that, as $n \to \infty$,

$$\sqrt{n}(\widehat{\theta}_{n,ML} - \theta) \xrightarrow{d} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, I(\theta)^{-1} \right) .$$

24) Let $X_1, \ldots, X_n \sim \mathcal{N}(\theta, 1)$ be independent, where $\theta \in \mathbb{R}$. However, only realizations of the random variables $Y_1, \ldots, Y_n$ are observed, where

$$Y_i = \begin{cases} 1, & \text{if } X_i \leq 0, \\ 0, & \text{if } X_i > 0 \end{cases} .$$

Compute the maximum likelihood estimator based on $Y_1, \ldots, Y_n$. Is this estimator consistent?

25) Show that the Hellinger distance and the Hellinger affinity do not depend on the choice of a dominating σ-finite measure μ.

Hint: See the proof of Lemma 1.3.1.