Mathematical Statistics, Winter term 2018/19
Problem sheet 11

33) Let $X_1, \ldots, X_n$ be i.i.d. with $X_i \sim \text{Uniform}([a,b])$, $\theta = \binom{a}{b} \in \Theta := \{\binom{x}{y}\colon\ -\infty < x < y < \infty\}$.

Find a sufficient statistic T with values in $\mathbb{R}^2$.

34) Let $(X, \Omega, \mathcal{A}, \{P_\theta\colon\ \theta \in \{\theta_0, \theta_1\}\})$ be a statistical experiment, where $P_\theta^X = \text{Bin}(n, \theta)$ and $\theta_0 \neq \theta_1$.

Show that $P_{\theta_0}(\varphi(X) = 1)$ and $P_{\theta_1}(\varphi(X) = 0)$ cannot be minimized simultaneously if $\{\theta_0, \theta_1\} \neq \{0, 1\}$.

35) Let $(X, \Omega, \mathcal{A}, \{P_\theta\colon\ \theta \in \{\theta_0, \theta_1\}\})$ be a statistical experiment, where $P_{\theta_0}^X$ and $P_{\theta_1}^X$ have respective densities p_{θ_0} and p_{θ_1} w.r.t. a σ-finite measure μ. A test φ of $H_0\colon\ \theta = \theta_0$ vs. $H_1\colon\ \theta = \theta_1$ has the form

$$\varphi(x) = \begin{cases} 1, & \text{if } p_{\theta_1}(x) > c p_{\theta_0}(x), \\ \gamma, & \text{if } p_{\theta_1}(x) = c p_{\theta_0}(x), \\ 0, & \text{if } p_{\theta_1}(x) < c p_{\theta_0}(x) \end{cases},$$

where $c \geq 0$ and $\gamma \in [0,1]$, and it holds that

$$E_{\theta_0}\varphi(X) = \alpha.$$

Show that φ is a most powerful test of H_0 vs. H_1 in the class of all (non-randomized and randomized) tests.