**Mathematical Statistics, Winter term 2018/19**
Problem sheet 12

36) Let $X_1, \dots, X_n$ be i.i.d. with $X_i \sim \text{Bin}(1, \theta)$, where $\theta \in \Theta := (0,1)$.
Find a (possibly randomized) test $\varphi$ which minimizes

$$\beta \int \varphi(x)\, dP^X_{\theta_0}(x) \;+\; (1-\beta) \int (1-\varphi(x))\, dP^X_{\theta_1}(x),$$

for some $\beta \in [0,1]$.

37) Let $X_1, \dots, X_n$ be independent random variables with $X_i \sim \text{Uniform}([\theta, \theta+1])$.
Find a most powerful test of size $\alpha > 0$ for the problem

$$H_0\colon \quad \theta = 0 \qquad \text{vs.} \qquad H_1\colon \quad \theta = c,$$

where $c \in (0,1)$.

38) Let $X_1, \dots, X_n$ be independent random variables with $X_i \sim \mathcal{N}(\theta, 1)$, $i = 1, \dots, n$. Consider the problem of testing the following hypotheses.

$$H_0\colon \quad \theta = \theta_0 \qquad \text{vs.} \qquad H_1\colon \quad \theta = \theta_1,$$

where $\theta_0 < \theta_1$.

How large must the sample size $n$ be in order that the probabilities of type 1 and type 2 errors are both not greater than 0.05?

*Hint: It holds that* $\Phi^{-1}(0.95) \approx 1.64$.