39) (i) Show that the family of distributions $\{\mathrm{Bin}(n, \theta)\colon \theta \in (0, 1)\}$ has a monotone likelihood ratio.

(ii) For $X \sim \mathrm{Bin}(n, \theta)$, construct a UMP test of size $\alpha \in (0, 1)$ for the problem

$$H_0\colon \quad \theta \leq 1/2 \qquad \text{vs.} \qquad H_1\colon \quad \theta > 1/2.$$

40) Assume that a random variable X has a density $p_\theta = dP_\theta^X/d\lambda$ with $p_\theta(x) = \frac{1}{2}e^{-|x-\theta|}$, where $\theta \in \mathbb{R}$.

(i) Does the family $\{P_\theta^X\colon \theta \in \mathbb{R}\}$ have a monotone likelihood ratio?

(ii) Construct a UMP test of size $\alpha \in (0, 1)$ for

$$H_0\colon \quad \theta \leq \theta_0 \qquad \text{vs.} \qquad H_1\colon \quad \theta > \theta_0.$$

(iii) Compute the p-value $\widehat{\alpha}(X)$.

41) Let $X_1, \ldots, X_1, Y_1, \ldots, Y_n$ be independent random variables, where $X_i \sim \mathcal{N}(\theta_1, 1)$ and $Y_i \sim \mathcal{N}(\theta_2, 1)$.

Find a likelihood ratio test of size $\alpha > 0$ for

$$H_0\colon \quad \theta_1 = \theta_2 \qquad \text{vs.} \qquad \theta_1 \neq \theta_2.$$