Mathematical Statistics, Winter term 2018/19

Problem sheet 14

42) Assume that Z has a t-distribution with n degrees of freedom.

Show that

$$P(Z \leq t) = P(-Z \leq t) \qquad \forall t \in \mathbb{R}.$$

Hint: Use the fact that, for $X \sim \mathcal{N}(0,1)$, $P(X \leq u) = P(-X \leq u)$ *holds for all* $u \in \mathbb{R}$.

42) Show that the one-tailed t-test is unbiased.

43) Let X be an $(n \times k)$-matrix with $rank(X) = k$.

(i) Show that $X^T X$ is a regular matrix.

(ii) Show that $X(X^T X)^{-1} X^T$ is the (unique) projection matrix onto the subspace $\Theta = \{X\beta \colon \beta \in \mathbb{R}^k\}$.