NAME: TOTAL:

Final Exam, MATH 350, Spring 2018 - Linde

- Points deduced for poorly presented solutions or untidy work.
- Show all important steps. Correct answers with incorrect or incomplete arguments to support them will receive no credit.

Distribution of points:

Problem 1 : 10 + 5* Points
Problem 2 : 10 Points
Problem 3 : 15 Points
Problem 4 : 10 Points
Problem 5 : 15 Points
Problem 6 : 15 Points
Problem 7 : 10 Points
Problem 8 : 15 Points

Sum: 100 + 5* Points

1. 20 people each randomly choose a number from 1 to 10. Each person's choice is independent. All 10 numbers are equally likely to be chosen.

   (a) How many different results can be observed if it is not known who of the 20 persons had chosen which number?

   (b) Find the probability that the number 1 was chosen twice, number 2 three times and number 3 once.

   (c) ($\star$) How likely is it that "1" was chosen exactly as often as "2" ?

2. Roll two dice 72 times. How likely is it to get a pair of 6's at least twice? Give the precise value as well as the one by using the Poisson approximation.

3. In an urn are 2 white balls and 4 red ones. Choose randomly and with replacement 3 balls. Let X be the number of the observed white balls.

   (a) Determine the **probability mass** as well as the **distribution function** of X.

   (b) Evaluate $\mathbb{E}X$ and $\mathbb{V}X$.

4. Two players, say player A and B, roll successively two fair dice. Winner is who first gets two even numbers. Player A starts, then B follows etc.

   (a) How likely is it[1] that A wins?

   (b) Find the probability that there is a winner (strictly) before A rolls the two dice the third time.

[1] Give the value as fraction.

5. Humans may have four different blood types, namely A, 0, B and AB. The distribution of these types are as follows: 45% of humans have group A, 40% group 0, 10% group B, and only 5% of the population possess type AB.

   Furthermore, humans may have either a positive or a negative rhesus factor. For example, 10% of humans with blood group A have a negative rhesus factor, 5% of those with group 0, 10% of people with group B and, finally, 20% with blood group AB possess a negative rhesus factor.

   (a) How likely is it that a randomly chosen person has a negative rhesus factor?

   (b) Suppose the chosen person has a negative rhesus factor. What is the probability that he has blood group A, 0, B or AB ?

6. There are 10 students in a lecture room: 6 are female and 4 are male. Choose randomly 3 of the 10 students (without replacement). Then the events A and B are defined as follows:

$$A = \{\text{There are students of different gender among the chosen three}\}$$

and

$$B = \{\text{At most one of the chosen students is female}\}.$$

Evaluate $\mathbb{P}(A)$, $\mathbb{P}(A|B)$ and $\mathbb{P}(A|B^c)$.

7. Suppose two **independent** events A and B satisfy

$$\mathbb{P}(A) = 0.6 \quad \text{and} \quad \mathbb{P}(A \cup B) = 0.8\,.$$

Find $\mathbb{P}(B)$, $\mathbb{P}(A \setminus B)$, $\mathbb{P}(B \setminus A)$, $\mathbb{P}(A^c \cup B^c)$ and $\mathbb{P}(A^c \cap B^c)$.

8. Suppose a random variable X has the distribution function F with

$$F(t) = \begin{cases} 0 & : \quad t < 1 \\ 1 - t^{-3} & : \ 1 \leq t < \infty \end{cases}$$

(a) Determine the density function of X.

(b) Evaluate $\mathbb{P}\{0.5 \leq X \leq 2\}$ and $\mathbb{P}\{X > 4\}$.

(c) Find $\mathbb{E}X$ and $\mathbb{V}X$.