Wahrscheinlichkeitstheorie und Statistik

Sommersemester 2020
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Hinweise und Musterlösung 3. Übungsblatt

Musterlösungen

Aufgabe 1.

(a) Code: x[t] /. DSolve[{x'[t] == -x[t], x[0] == 2}, x, t]
Lösung der DGL: $2e^{-t}$

(b) Code: x[t] /. DSolve[{x'[t] == 2 x[t] + t, x[0] == 1}, x, t]
Lösung der DGL: $\frac{1}{4}(-1 + 5e^{2t} - 2t)$

(c) Code: x[t] /. DSolve[{x'[t] == x[t] + Exp[t], x[0] == 3}, x, t]
Lösung der DGL: $e^t(3 + t)$

(d) Code: x[t] /. DSolve[{x'[t] == t^2 x[t] + t^2, x[0] == 0}, x, t]
Lösung der DGL: $e^{\frac{t^3}{3}} - 1$

Aufgabe 2. (a) Code: z = x[t] /. DSolve[{x″[t] + x[t] == 1, x[0] == 1, x′[3] == 2}, x, t]
Lösung der DGL: $1 + \frac{2\sin[t]}{\cos[3]}$

(b) Code: z = x[t] /. DSolve[{x"[t] + x[t] == 1, x[0] == 1, x[3] == 2}, x, t]
Lösung der DGL: $1 + \frac{\sin[t]}{\sin[3]}$

(c) Code: z = x[t] /. DSolve[{x"[t] + 2 x'[t] + 3 == Sin[t], x[0] == 2, x[4] == 0}, x, t]
Lösung der DGL:

$$-\left(\frac{1}{10}(-1+e^8)\right)e^{-2t}(36e^8 + 24e^{2t} - 60e^{8+2t} - 15e^{2t}t + 15e^{8+2t}t$$
$$+4e^8\cos[4] - 4e^{8+2t}\cos[4] - 4e^{2t}\cos[t] + 4e^{8+2t}\cos[t]$$
$$+2e^8\sin[4] - 2e^{8+2t}\sin[4] - 2e^{2t}\sin[t] + 2e^{8+2t}\sin[t]\big)$$

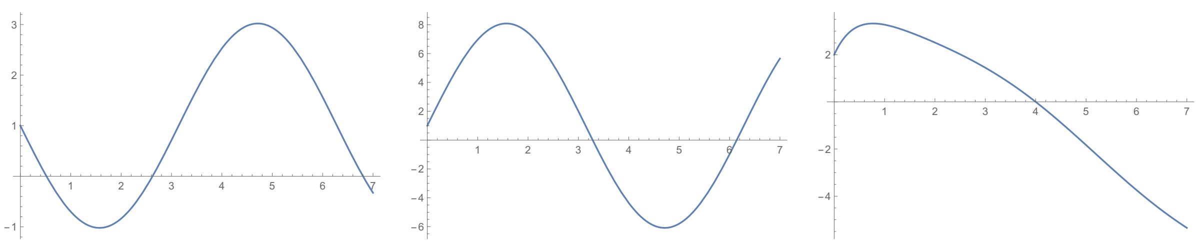


Abbildung 1: Links: (a) Mitte: (b) Rechts: (c)

Aufgabe 3. (a) Yule-Prozess: Setze die Werte für i, l durch z.B. l = 0.5; i = 1;
Danach folgt der Code

```
p1 = x[t] /. DSolve[ {x'[t] == - l (1 + i - 1) x[t], x[0] == 1 }, x[t], t][[1]]
p2 = x[t] /. DSolve[ {x'[t] == - l (2 + i - 1) x[t] + l (1 + i - 1) p1, x[0] == 0 }, x[t], t][[1]]
p3 = x[t] /. DSolve[ {x'[t] == - l (3 + i - 1) x[t] + l (2 + i - 1) p2, x[0] == 0 }, x[t], t][[1]]
p4 = x[t] /. DSolve[ {x'[t] == - l (4 + i - 1) x[t] + l (3 + i - 1) p3, x[0] == 0 }, x[t], t][[1]]
p5 = x[t] /. DSolve[ {x'[t] == - l (5 + i - 1) x[t] + l (4 + i - 1) p4, x[0] == 0 }, x[t], t][[1]]
```

und das Plotten durch

```
Plot[{p1, p2, p3, p4, p5, p1 + p2 + p3 + p4 + p5}, {t, 0, 5}, PlotRange -> {0, 1}]
```

Lösung der DGL im Allgemeinen:

$$p_k(t) = \binom{i+k-2}{k-1} e^{-lit}(1-e^{-lt})^{k-1}$$

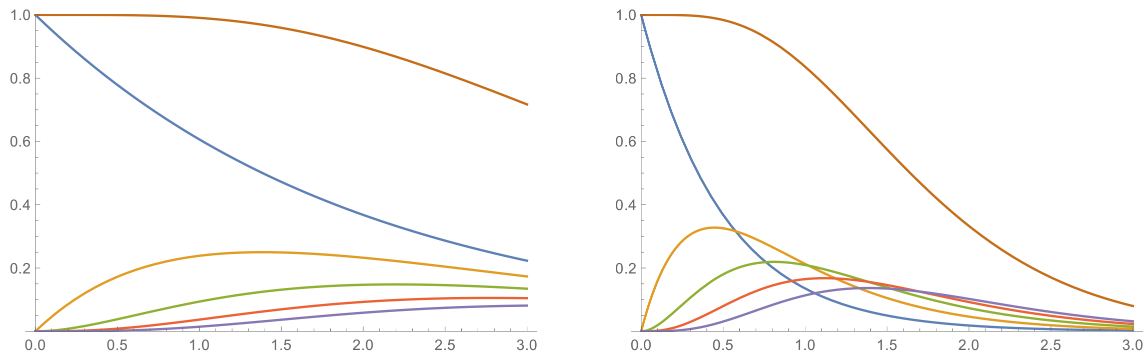


Abbildung 2: Links: l=0.5, i=1, Rechts: l=0.5, i=4

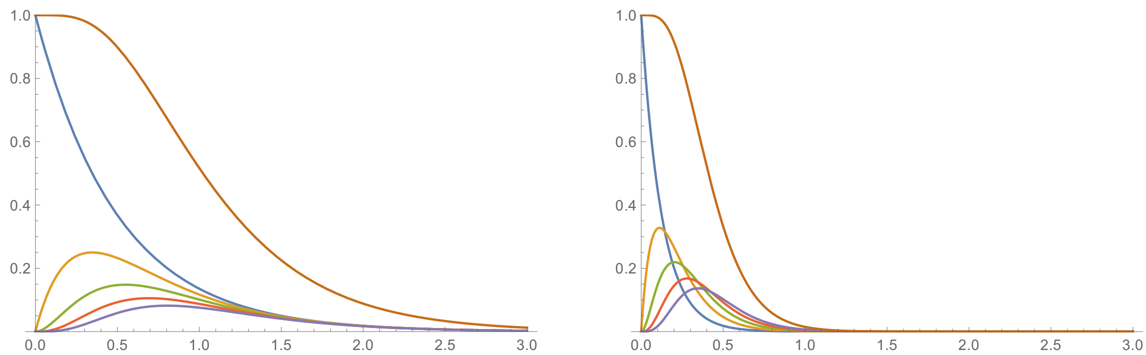
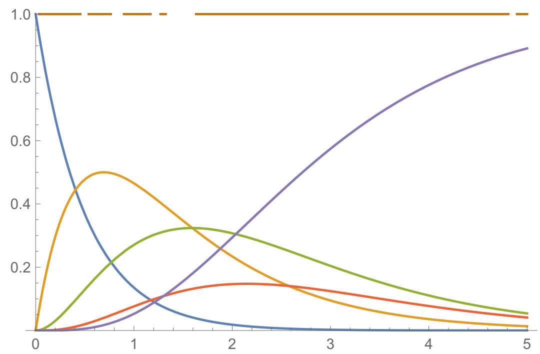


Abbildung 3: Links: l=2, i=1, Rechts: l=2, i=4

(b) Geburseprozess:

```
l = {2, 1, 1, 2, 0, 0};
p1 = x[t] /. DSolve[ {x'[t] == - l[[1]] x[t], x[0] == 1 }, x[t], t][[1]]
p2 = x[t] /. DSolve[ {x'[t] == - l[[2]] x[t] + l[[1]] p1, x[0] == 0 }, x[t], t][[1]]
p3 = x[t] /. DSolve[ {x'[t] == - l[[3]] x[t] + l[[2]] p2, x[0] == 0 }, x[t], t][[1]]
p4 = x[t] /. DSolve[ {x'[t] == - l[[4]] x[t] + l[[3]] p3, x[0] == 0 }, x[t], t][[1]]
p5 = x[t] /. DSolve[ {x'[t] == - l[[5]] x[t] + l[[4]] p4, x[0] == 0 }, x[t], t][[1]]
p6 = x[t] /. DSolve[ {x'[t] == - l[[6]] x[t] + l[[5]] p5, x[0] == 0 }, x[t], t][[1]]
Plot[{p1, p2, p3, p4, p5, p1 + p2 + p3 + p4 + p5}, {t, 0, 5}, PlotRange -> {0, 1}]
```

Explizite Lösung: $p_1(t) = e^{-2t}$, $p_2(t) = 2e^{-t}(1-e^{-t})$, $p_3(t) = 2e^{-t}(e^{-t}-1+t)$, $p_4(t) = 2e^{-t}(2e^{-t}-2+te^{-t}+t)$, $p_5(t) = e^{-t}(4-5e^{-t}+e^{t}-2te^{-t}-4t)$, $p_6(t) = 0$



(c) Todesprozess:
l = {1, 2, 3, 4, 5};
p1 = x[t] /. DSolve[{x'[t] == - l[[1]] x[t], x[0] == 1 }, x[t], t][[1]]
p2 = x[t] /. DSolve[{x'[t] == - l[[2]] x[t] + l[[1]] p1, x[0] == 0 }, x[t], t][[1]]
p3 = x[t] /. DSolve[{x'[t] == - l[[3]] x[t] + l[[2]] p2, x[0] == 0 }, x[t], t][[1]]
p4 = x[t] /. DSolve[{x'[t] == - l[[4]] x[t] + l[[3]] p3, x[0] == 0 }, x[t], t][[1]]
p5 = x[t] /. DSolve[{x'[t] == - l[[5]] x[t] + l[[4]] p4, x[0] == 0 }, x[t], t][[1]]
p6 = x[t] /. DSolve[{x'[t] == l[[5]] p5, x[0] == 0 }, x[t], t][[1]]
Plot[{p1, p2, p3, p4, p5, p6, p1 + p2 + p3 + p4 + p5 + p6}, {t, 0, 4}, PlotRange -> {0, 1}]
Explizite Lösung: $p_1(t) = e^{-t}$, $p_2(t) = e^{-t}(1-e^{-t})$, $p_3(t) = e^{-t}(1-e^{-t})^2$, $p_4(t) = e^{-t}(1-e^{-t})^3$, $p_5(t) = e^{-t}(1-e^{-t})^4$, $p_6(t) = (1-e^{-t})^5$