Mathematical Statistics, Winter semester 2020/21

Problem sheet 2

4) Suppose that

$$Y_i = \theta + \varepsilon_i, \qquad i = 1, \ldots, n,$$

holds for some $\theta \in \Theta := \mathbb{R}$, where $\varepsilon_1, \ldots, \varepsilon_n$ are independent random variables such that $E\varepsilon_i = 0$ and $\mathrm{var}(\varepsilon_i) = \sigma_i^2 > 0$ for $i = 1, \ldots, n$.

Compute the best linear estimator of θ.

5) Let $X_1, \ldots, X_n \sim \mathrm{Bin}(1, \theta)$ be independent random variables, $\theta \in \Theta := [0, 1]$.

Compute the maximum likelihood estimator of θ.

6) Let $X_1, \ldots, X_n$ be i.i.d. with $X_i \sim \mathrm{Uniform}([\theta_1, \theta_2])$, where $-\infty < \theta_1 < \theta_2 < \infty$.

(i) Compute the moment estimator of $\theta = (\theta_1, \theta_2)^T$.

(ii) Compute the maximum likelihood estimator of θ.