Mathematical Statistics, Winter semester 2020/21

Problem sheet 3

7) Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with $P_\theta(X_i = 1) = \theta = 1 - P_\theta(X_i = 0)$, where $\theta \in \Theta = (0, 1)$.

   Show that there is no unbiased estimator $T = T(X_1, \ldots, X_n)$ of the parameter $g(\theta) = 1/\theta$.

8) Suppose that a realization of a random variable X is observed, $X \sim P_\theta$, where $\theta \in \Theta$. Suppose further that there exists some $\theta_0 \in \Theta$ such that $P_\theta \ll P_{\theta_0}$ $\forall \theta \in \Theta$, that is, $P_{\theta_0}(B) = 0$ implies $P_\theta(B) = 0$.

   Show that $T \equiv \theta_0$ is an admissible estimator of θ when the mean squared error is taken as a measure of performance.

9) Show that the Hellinger affinity, and therefore the Hellinger distance as well, do not depend on the choice of a dominating σ-finite measure μ.

   Hint: See the proof of Lemma 2.1.

10) Let $X \sim P_\theta = \text{Poisson}(\theta)$, where $\theta \in \Theta = (0, \infty)$.

    (i) Compute the Fisher information of the family $\{P_\theta\colon\ \theta \in \Theta\}$.

    (ii) Compute the mean squared error of the estimator $T(X) = X$ for the parameter θ.
    Hint: Compute first $E_\theta X$ and $E_\theta[X(X-1)]$.