Mathematical Statistics, Winter semester 2020/21
Problem sheet 4

11) Compute the Fisher information number $I(\theta)$ of the family $\{N(\theta, \sigma^2)\colon\ \theta \in \mathbb{R}\}$. ($\sigma^2 > 0$ is fixed.)

12) Let $X_1, \dots, X_n$ be i.i.d. with $P_\theta(X_i = 1) = \theta = 1 - P_\theta(X_i = 0)$, where $\theta \in \Theta = (0, 1)$. Show with the aid of Proposition 2.10 that $\bar{X}_n = n^{-1} \sum_{i=1}^{n} X_i$ is admissible (w.r.t. the mean squared error) in the class of all estimators.

13) Suppose that a realization of $X \sim P_\theta := \mathrm{Bin}(\theta, p)$ is observed, where $\theta \in \Theta := \mathbb{N}$ and $p \in (0, 1)$ is known. Let $\pi = \mathrm{Poisson}(\lambda)$, $\lambda > 0$, be the prior distribution for θ.

    (i) Find the posterior distribution of θ given $X = k$.

    (ii) Suppose that the mean squared error is chosen as measure of the performance of an estimator. Compute the Bayes estimator.