

Mathematical Statistics, Winter semester 2020/21
Problem sheet 6

- 17) (i) Show that the family of distributions $\{\text{Bin}(n, \theta): \theta \in (0, 1)\}$ has a monotone likelihood ratio.
(ii) For $X \sim \text{Bin}(n, \theta)$, construct a UMP test of size $\alpha \in (0, 1)$ for the problem

$$H_0: \theta \leq 1/2 \quad \text{vs.} \quad H_1: \theta > 1/2.$$

- 18) Assume that a random variable X has a density $p_\theta = dP_\theta^X/d\lambda$ with $p_\theta(x) = \frac{1}{2}e^{-|x-\theta|}$, where $\theta \in \mathbb{R}$.

- (i) Does the family $\{P_\theta^X: \theta \in \mathbb{R}\}$ have a monotone likelihood ratio?
(ii) Construct a UMP test of size $\alpha \in (0, 1)$ for

$$H_0: \theta \leq \theta_0 \quad \text{vs.} \quad H_1: \theta > \theta_0.$$

- 19) Let $X_1, \dots, X_n, Y_1, \dots, Y_n$ be independent random variables, where $X_i \sim \mathcal{N}(\theta_1, 1)$ and $Y_i \sim \mathcal{N}(\theta_2, 1)$.

Find a likelihood ratio test of size $\alpha > 0$ for

$$H_0: \theta_1 = \theta_2 \quad \text{vs.} \quad \theta_1 \neq \theta_2.$$