

**Measure Theory, Winter semester 2021/22**  
Problem sheet 4

- 7) Consider the measure space  $(\mathbb{R}, \mathcal{B}, \delta_0)$ , where  $\delta_0$  denotes the Dirac measure concentrated at 0, i.e.

$$\delta_0(A) = \begin{cases} 1 & \text{if } 0 \in A, \\ 0 & \text{if } 0 \notin A. \end{cases}$$

Find the completion of  $\mathcal{B}$ .

- 8) Show that there is a Lebesgue measurable subset of  $\mathbb{R}^2$  whose projection on  $\mathbb{R}$  under the map  $(x, y) \mapsto x$  is not Lebesgue measurable.