Measure Theory, Winter semester 2021/22
Problem sheet 5

9) Let $(\Omega, \mathcal{A})$ be a measurable space, and let $f, g \colon \Omega \to \mathbb{R}$ be $(\mathcal{A}-\mathcal{B})$-measurable functions.
Show that the sets $\{\omega \in \Omega \colon f(\omega) < g(\omega)\}$ and $\{\omega \in \Omega \colon f(\omega) = g(\omega)\}$ belong to $\mathcal{A}$.

10) Suppose that $f \colon \mathbb{R} \to \mathbb{R}$ is differentiable everywhere on $\mathbb{R}$.
Show that f' is $(\mathcal{B} - \mathcal{B})$-measurable.

11) Let $(\Omega, \mathcal{A}, \mu)$ be a measure space, and let $f \colon \Omega \to \{0, 1, 2, \ldots\}$ be a non-negative integer-valued $(\mathcal{A} - \mathcal{B})$-measurable function.
Show that $\int_\Omega f \, d\mu = \sum_{n=1}^{\infty} \mu(\{\omega \colon f(\omega) \geq n\})$.