Mathematical Statistics, Winter semester 2021/22

Problem sheet 4

10) Let $X \sim P_\theta = \text{Poisson}(\theta)$, where $\theta \in \Theta = (0, \infty)$.

(i) Compute the Fisher information of the family $\{P_\theta\colon\ \theta \in \Theta\}$.

(ii) Compute the mean squared error of the estimator $T(X) = X$ for the parameter θ. *Hint: Compute first $E_\theta X$ and $E_\theta[X(X-1)]$.*

11) Compute the Fisher information number $I(\theta)$ of the family $\{N(\theta, \sigma^2)\colon\ \theta \in \mathbb{R}\}$. ($\sigma^2 > 0$ is fixed.)

12) Let $X_1, \ldots, X_n$ be independent random variables, $X_i \sim Uniform[\theta_1, \theta_2]$, where $\theta = \binom{\theta_1}{\theta_2} \in \Theta := \{\binom{a}{b}\colon -\infty < a < b < \infty\}$.

Show that $T(X_1, \ldots, X_n) = (X_{n:1}, X_{n:n})$, where $X_{n:1} = \min\{X_1, \ldots, X_n\}$ and $X_{n:n} = \max\{X_1, \ldots, X_n\}$, is a sufficient statistic for $\theta \in \Theta$.