

Mathematical Statistics, Winter semester 2021/22

Problem sheet 5

13) Let  $X_1, \dots, X_n$  be i.i.d. with  $P_\theta(X_i = 1) = \theta = 1 - P_\theta(X_i = 0)$ , where  $\theta \in \Theta = (0, 1)$ .

Show with the aid of Proposition 2.9 (Lecture notes, page 46) that  $\bar{X}_n = n^{-1} \sum_{i=1}^n X_i$  is admissible (w.r.t. the mean squared error) in the class of all estimators.

14) Suppose that a realization of  $X \sim P_\theta := \text{Bin}(\theta, p)$  is observed, where  $\theta \in \Theta := \mathbb{N}$  and  $p \in (0, 1)$  is known. Let  $\pi = \text{Poisson}(\lambda)$ ,  $\lambda > 0$ , be the prior distribution for  $\theta$ .

(i) Find the posterior distribution of  $\theta$  given  $X = k$ .

(ii) Suppose that the mean squared error is chosen as measure of the performance of an estimator. Compute the Bayes estimator.