Mathematical Statistics, Winter semester 2021/22
Problem sheet 6

15) Let $X_1, \dots, X_n$ be i.i.d. with $X_i \sim \text{Bin}(1, \theta)$, where $\theta \in \Theta := \{\theta_0, \theta_1\} \subseteq (0,1)$, $\theta_0 \neq \theta_1$. For $\beta \in [0,1]$, find a (possibly randomized) test φ which minimizes

$$\beta\, E_{\theta_0}[\varphi(X)] \,+\, (1-\beta)\, E_{\theta_1}[1-\varphi(X)].$$

$(X = (X_1, \dots, X_n)^T)$

16) Let $X_1, \dots, X_n$ be independent random variables with $X_i \sim \mathcal{N}(\theta, 1)$, $i = 1, \dots, n$. Consider the problem of testing the following hypotheses.

$$H_0\colon \quad \theta = \theta_0 \qquad \text{vs.} \qquad H_1\colon \quad \theta = \theta_1,$$

where $\theta_0 < \theta_1$.

How large must the sample size n be in order that the probabilities of type I and type II errors are both not greater than 0.05?

Hint: It holds that $\Phi^{-1}(0.95) \approx 1.64$.

17) (i) Show that the family of distributions $\{\text{Bin}(n, \theta)\colon \theta \in (0,1)\}$ has a monotone likelihood ratio.

(ii) For $X \sim \text{Bin}(n, \theta)$, construct a UMP test of size $\alpha \in (0,1)$ for the problem

$$H_0\colon \quad \theta \le 1/2 \qquad \text{vs.} \qquad H_1\colon \quad \theta > 1/2.$$