Fachbereich Grundlagenwissenschaften
PD Dr. Hartmut Menzer
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Mathematik für Business Administration

Übungsaufgaben

Serie 7: Funktionen mit mehreren Veränderlichen - Lösungshinweise

1. a) $f_x = 2x\ln y - \frac{y^2}{x} + 1, \quad f_{xx} = 2\ln y + \frac{y^2}{x^2},$
   $f_y = \frac{x^2}{y} - 2y\ln x, \quad f_{yy} = -\frac{x^2}{y^2} - 2\ln x, \qquad f_{xy} = \frac{2x}{y} - \frac{2y}{x}$

   b) $f_x = \mathrm{e}^{x-y^2}(x^2+2x) + \frac{2}{x} + 1, \quad f_{xx} = \mathrm{e}^{x-y^2}(x^2+4x+2) - \frac{2}{x^2},$
   $f_y = -2yx^2\mathrm{e}^{x-y^2}, \quad f_{yy} = \mathrm{e}^{x-y^2}(-2x^2+4x^2y^2), \qquad f_{xy} = -2y(x^2+2x)\mathrm{e}^{x-y^2}$

   c) $f_x = 3(x+3)^2(y+6)^2, \quad f_{xx} = 6(x+3)(y+6)^2,$
   $f_y = 2(x+3)^3(y+6), \quad f_{yy} = 2(x+3)^3, \qquad f_{xy} = 6(x+3)^2(y+6)$

   d) $f_x = -x(y-x^2)^{-\frac{1}{2}}, \quad f_{xx} = -(y-x^2)^{-\frac{1}{2}} - x^2(y-x^2)^{-\frac{3}{2}},$
   $f_y = \frac{1}{2}(y-x^2)^{-\frac{1}{2}}, \quad f_{yy} = -\frac{1}{4}(y-x^2)^{-\frac{3}{2}}, \qquad f_{xy} = \frac{1}{2}x(y-x^2)^{-\frac{3}{2}}$

   e) $f_x = 1 - \frac{y}{x^2}, \quad f_{xx} = \frac{2y}{x^3}, \qquad f_y = \frac{1}{x}, \quad f_{yy} = 0, \qquad f_{xy} = -\frac{1}{x^2}$

   f) $f_x = 16x + 4x^3 - 8xy^2, \quad f_{xx} = 16 + 12x^2 - 8y^2,$
   $f_y = -32y - 8yx^2 + 16y^3, \quad f_{yy} = -32 - 8x^2 + 48y^2, \qquad f_{xy} = -16xy$

   g) $f_x = \ln(\frac{y^2}{2x+1}) - \frac{2x}{2x+1}, \quad f_{xx} = -4\frac{x+1}{(2x+1)^2}, \qquad f_y = \frac{2x}{y}, \quad f_{yy} = -\frac{2x}{y^2}, \qquad f_{xy} = \frac{2}{y}$

   h) $f_x = 3x^2yz^2 + 4xy^2z, \quad f_{xx} = 6xyz^2 + 4y^2z, \qquad f_y = x^3z^2 + 4x^2yz, \quad f_{yy} = 4x^2z,$
   $f_z = 2x^3yz + 2x^2y^2 + 15z^2, \quad f_{zz} = 2x^3y + 30z,$
   $f_{xy} = 3x^2z^2 + 8xyz, \quad f_{xz} = 6x^2yz + 4xy^2, \quad f_{yz} = 2x^3z + 4x^2y$

   i) $f_{x_1} = \mathrm{e}^{x_2-x_3} + \frac{1}{2}x_2(x_1x_2+x_3)^{-\frac{1}{2}} + (x_1+{x_3}^2)^{-1},$
   $f_{x_1x_1} = -\frac{1}{4}{x_2}^2(x_1x_2+x_3)^{-\frac{3}{2}} - (x_1+{x_3}^2)^{-2},$
   $f_{x_2} = x_1\mathrm{e}^{x_2-x_3} + \frac{1}{2}x_1(x_1x_2+x_3)^{-\frac{1}{2}}, \quad f_{x_2x_2} = x_1\mathrm{e}^{x_2-x_3} - \frac{1}{4}{x_1}^2(x_1x_2+x_3)^{-\frac{3}{2}},$
   $f_{x_3} = -x_1\mathrm{e}^{x_2-x_3} + 1 + \frac{1}{2}(x_1x_2+x_3)^{-\frac{1}{2}} + \frac{2x_3}{x_1+{x_3}^2},$
   $f_{x_3x_3} = x_1\mathrm{e}^{x_2-x_3} - \frac{1}{4}(x_1x_2+x_3)^{-\frac{3}{2}} + \frac{2x_1-2{x_3}^2}{(x_1+{x_3}^2)^2},$
   $f_{x_1x_2} = \mathrm{e}^{x_2-x_3} - \frac{1}{4}x_1x_2(x_1x_2+x_3)^{-\frac{3}{2}} + \frac{1}{2}(x_1x_2+x_3)^{-\frac{1}{2}},$
   $f_{x_1x_3} = -\mathrm{e}^{x_2-x_3} - \frac{1}{4}x_2(x_1x_2+x_3)^{-\frac{3}{2}} - \frac{2x_3}{(x_1+{x_3}^2)^2},$
   $f_{x_2x_3} = -x_1\mathrm{e}^{x_2-x_3} - \frac{1}{4}x_1(x_1x_2+x_3)^{-\frac{3}{2}}$

   j) $f_{x_j} = \frac{1}{n}, \quad j = 1,2,...,n, \quad f_{x_ix_j} = 0, \quad i,j = 1,2,...,n$

   k) $f_{x_j} = \frac{1}{n}(x_1x_2...x_n)^{\frac{1}{n}-1}(x_1x_2...x_{j-1}x_{j+1}...x_n) = \frac{1}{n}{x_j}^{\frac{1}{n}-1} \cdot \sqrt[n]{x_1 \cdot ... \cdot x_{j-1}x_{j+1} \cdot ... \cdot x_n},$
   $j = 1,2,...,n$.

2. a) Minimum in $(4,3)$, (Sattelpunkt in $(-4,3)$)
   b) Minimum in $(2,-5)$ und in $(-2,-5)$, (Sattelpunkt in $(0,-5)$)
   c) keine Extrempunkte
   d) Minimum in $(0,0)$, (Sattelpunkte in $(3,1)$, $(-3,1)$, $(3,-1)$, $(-3,-1)$)

3. Maximum in $(14,10)$

4. Maximum in $(6,66...\,;\,3,33...)$

5. Minimum in $(16, 9)$ ($y = -9$ entfällt)
6. $a = 70$, Maximum in $(12, 41)$, Gewinn $G(12, 41) = 1477$
7. 1%ige Erhöhung der Nutzensfunktion