



EMS Tracts in Mathematics, Vol. 6

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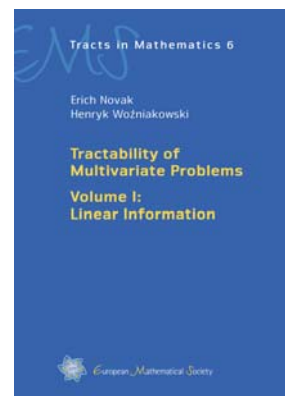
Tractability of Multivariate Problems Volume I: Linear Information

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Multivariate problems occur in many applications. These problems are defined on spaces of d -variate functions and d can be huge – in the hundreds or even in the thousands. Some high-dimensional problems can be solved efficiently to within ε , i.e., the cost increases polynomially in ε^{-1} and d . However, there are many multivariate problems for which even the minimal cost increases exponentially in d . This exponential dependence on d is called *intractability* or the *curse of dimensionality*.

This is the first of a three-volume set comprising a comprehensive study of the tractability of multivariate problems. It is devoted to algorithms using linear information consisting of arbitrary linear functionals. The theory for multivariate problems is developed in various settings: worst case, average case, randomized and probabilistic. A problem is tractable if its minimal cost is *not* exponential in ε^{-1} and d . There are various notions of tractability, depending on how we measure the lack of exponential dependence. For example, a problem is polynomially tractable if its minimal cost is polynomial in ε^{-1} and d . The study of tractability was initiated about 15 years ago. This is the first research monograph on this subject.

Many multivariate problems suffer from the curse of dimensionality when they are defined over classical (unweighted) spaces. But many practically important problems are solved today for huge d in a reasonable time. One of the most intriguing challenges of theory is to understand why this is possible. Multivariate problems may become tractable if they are defined over *weighted* spaces with properly decaying weights. In this case, all variables and groups of variables are moderated by weights. The main purpose of this book is to study weighted spaces and to obtain conditions on the weights that are necessary and sufficient to achieve various notions of tractability.

The book is of interest for researchers working in computational mathematics, especially in approximation of high-dimensional problems. It may be also suitable for graduate courses and seminars. The text concludes with a list of thirty open problems that can be good candidates for future tractability research.

Contents: Overview

Motivation for tractability studies · Notes and remarks

Twelve examples · Tractability in the worst case setting · Tractability in other settings · Open problems · Notes and remarks

Basic concepts and survey of IBC results · Complexity in the worst case setting · Basic results for linear problems in the worst case setting · Some results for different settings · Multivariate problems and tractability · Notes and remarks

Worst case setting · Linear problems defined over Hilbert spaces · Linear tensor product problems · Linear weighted tensor product problems · Other ways of obtaining linear weighted problems · Notes and remarks

Average case setting · Linear problems · Linear tensor product problems · Linear weighted tensor product problems · Notes and remarks

Randomized setting · Tractability of linear problems for Λ^{all}

Generalized tractability · Motivation of generalized tractability · Linear tensor product problems · Restricted tractability domain · Unrestricted tractability domain · Comparison · Notes and remarks

Appendix A. Reproducing kernel Hilbert spaces of Sobolev type · Korobov spaces · Sobolev spaces

Appendix B. Gaussian measures · Gaussian measures on Banach spaces · Gaussian measures and reproducing kernel Hilbert spaces

Appendix C. List of open problems

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