$2.M_{22}.2$ in characteristic 3

- $F[2.M_{22}.2]$ has 13 blocks; the faithful ones of positive defect are the block B_4 whose defect groups are elementary abelian of order 9, and the blocks B_5 and B_6 of defect 1
- $P \in \mathrm{Syl}_3(2.M_{22})$ is a defect group of B_4; $N_{2.M_{22}.2}(P)$ has order 288 and is isomorphic to an extension $N_{2.M_{22}}(P).2$
- The Brauer correspondent of B_4 is denoted by b_4, and it contains the simple $F[N_{2.M_{22}.2}(P)]$-modules 1_5, 1_6, 1_5^*, 1_6^*, 2_3, 2_5, 2_6. The modules 2_5 and 2_6 restrict irreducibly to $N_{2.M_{22}}(P)$, and the restriction of 2_3 to $N_{2.M_{22}}(P)$ splits into the direct sum of two one-dimensional modules.
- $C \leq 2.M_{22}$ is a defect group of both B_5 and B_6; $N_{2.M_{22}.2}(C)$ has order 288 and is isomorphic to an extension $N_{2.M_{22}}(C).2 \cong (\mathfrak{S}_4 \times C_2).2$. We identify $N_{2.M_{22}}(C)$ with $\mathfrak{S}_4 \times C_2$, and view it as the inner direct product of the subgroups $H_1 := \{(x, 1) \mid x \in \mathfrak{S}_4\} \cong \mathfrak{S}_4$ and C_2. Then the Brauer correspondent b_5 of B_5 contains simple modules 3_3 and 3_4, and the Brauer correspondent b_6 of B_6 contains simple modules 3_5 and 3_6 such that
$$\mathrm{Res}^{N_{2.M_{22}.2}(C)}_{N_{2.M_{22}}(C)}(3_3) \cong \mathrm{Inf}^{N_{2.M_{22}}(C)}_{C}(D^{(2,1^2)} \boxtimes \mathbf{sgn}) \cong \mathrm{Res}^{N_{2.M_{22}.2}(C)}_{N_{2.M_{22}}(C)}(3_5)$$
and
$$\mathrm{Res}^{N_{2.M_{22}.2}(C)}_{N_{2.M_{22}}(C)}(3_4) \cong \mathrm{Inf}^{N_{2.M_{22}}(C)}_{C}(D^{(3,1)} \boxtimes \mathbf{sgn}) \cong \mathrm{Res}^{N_{2.M_{22}.2}(C)}_{N_{2.M_{22}}(C)}(3_6).$$
Here **sgn** denotes the alternating FC_2-module.
- The simple $F[2.M_{22}.2]$-modules belonging to the blocks B_4, B_5 and B_6, respectively, have the defect groups of their blocks as vertices. Moreover all of these modules have trivial sources, and, for an appropriate labelling, their Green correspondents are as follows:

block	B_4						
module	$D(10_1)$	$D(10_1)^*$	$D(10_2)$	$D(10_2)^*$	$D(56_1)$	$D(56_2)$	$D(308)$
Green	1_5	1_5^*	1_6	1_6^*	2_5	2_6	2_3

block	B_5		B_6	
module	$D(120_1)$	$D(210_4)$	$D(120_2)$	$D(210_3)$
Green	3_4	3_3	3_6	3_5

- $2.M_{22}.2 \longrightarrow M_{22}{:}2$ the natural epimorphism, c and d standard generators of $M_{22}{:}2$
- standard generators of $2.M_{22}.2$ are preimages C of c and D od d such that CD has order 11
- $g := CCDCDCDCDDCDD$ and $h := g^3$ are non-conjugate element of order 14
- $x := CDCDD$ is an element of order 10

- $y := (CDD)^3$ has order 2, its conjugacy class lies above conjugacy class $2B$ of $M_{22}{:}2$
- $\zeta := \exp(2\pi i/8)$, $^{-} : \mathbb{Z}[\zeta] \longrightarrow \mathbb{F}_9$, $\zeta \longmapsto \mathbb{F}_9.1$ the lifting map

Then the modular character of the simple modules belonging to B_4 are as follows:

module	g	h	x
$D(10_1)_{2.22.2}$	$\mathbb{F}_9.1+2=\overline{b7**}$	$2\mathbb{F}_9.1=\overline{b7}$	0
$D(10_2)_{2.22.2}$	$\mathbb{F}_9.1^3=\overline{-b7**}$	$\mathbb{F}_9.1=\overline{-b7}$	0
$D(10_1)^*_{2.22.2}$	$\overline{b7}$	$\overline{b7**}$	0
$D(10_1)^*_{2.22.2}$	$\overline{-b7}$	$\overline{-b7**}$	0
$D(56_1)_{2.22.2}$	0	0	$-(\mathbb{F}_9.1+1)=\overline{-r5}$
$D(56_2)_{2.22.2}$	0	0	$\mathbb{F}_9.1+1=\overline{r5}$

Furthermore, the modular character of $D(120_2)_{2.22.2}$ on y is $8 \equiv 2 \pmod 3$, and that of $D(210_4)_{2.22.2}$ on y is $1 \equiv 28 \pmod 3$.