$2.M_{22}$ in characteristic 3

- $F[2.M_{22}]$ has nine blocks; the faithful ones of positive defect are the block B_3 of defect 2 with elementary abelian defect groups, and the block B_4 of defect 1.
- $P \in \mathrm{Syl}_3(2.M_{22})$; $N_{2.M_{22}}(P)$ has order 144, and if $Q \in \mathrm{Syl}_3(N_{2.M_{22}}(P))$ then $Q/Z(2.M_{22})$ is isomorphic to the quaternion group of order 8.
- The Brauer correspondent of the block B_3 is denoted by b_3, and it contains the simple modules 1_5, $1_6 = 1_5^*$, 2_1, 1_7, $1_8 = 1_7^*$.
- $C \cong C_3$ denotes a defect group of the block B_4; $N_{2.M_{22}}(C)$ has order 144, and $N_{2.M_{22}}(C)/C \cong \mathfrak{S}_4 \times C_2$. We identify $N_{2.M_{22}}(C)/C$ with $\mathfrak{S}_4 \times C_2$ and view it as the inner direct product of the subgroups $H_1 := \{(x,1) \mid x \in \mathfrak{S}_4\}$ and C_2.
- The simple $F[2.M_{22}]$-modules belonging to the blocks B_3 and B_4, respectively, have the defect groups of their blocks as vertices. Furthermore, all of these simple modules have trivial sources and the following Green correspondents:

block	B_3				
module	$D(10)_{2.22}$	$D(10)^*_{2.22}$	$D(56)_{2.22}$	$D(154)_{2.22}$	$D(154)^*_{2.22}$
Green	1_5	$1_6 = 1_5^*$	2_1	1_7	$1_8 = 1_7^*$

block	B_4	
module	$D(120)_{2.22}$	$D(210_1)_{2.22}$
Green	$\mathrm{Inf}_C^{N_{2.M_{22}}(C)}(D^{(3,1)} \boxtimes \mathbf{sgn})$	$\mathrm{Inf}_C^{N_{2.M_{22}}(C)}(D^{(2,1^2)} \boxtimes \mathbf{sgn})$

- $2.M_{22} \longrightarrow M_{22}$ the natural epimorphism, a and b standard generators of M_{22}
- standard generators of $2.M_{22}$ are preimages A of a and B of b such that A lies in class $+2A$ and B lies in class $-4A$
- representative for conjugacy class $7A$: $ABABABABBBABB$
- $g := ABABABBBABB$ is an element of order 8
- $\zeta := \exp(2\pi i/8)$, $^- : \mathbb{Z}[\zeta] \longrightarrow \mathbb{F}_9$, $\zeta \longmapsto \mathbb{F}_9.1$ the lifting map

module	conj. class	modular char. value	Brauer char.
$D(10)_{2.22}$	$7A$	$\mathbb{F}_9.1^5 = 2\mathbb{F}_9.1 = \overline{b7}$	φ_{11}
$D(154)_{2.22}$	g	$\mathbb{F}_9.1^2 = \mathbb{F}_9.1 + 1 = \overline{-2i}$	

Hence, for the simple modules belonging to B_3, we get:

$$D(10)_{2.22} \leftrightarrow \varphi_{11},\ D(10)^*_{2.22} \leftrightarrow \varphi_{12},\ D(56)_{2.22} \leftrightarrow \varphi_{13},\ D(154)_{2.22},\ D(154)^*_{2.22},$$

The Brauer character of $D(154)_{2.22}$ is either φ_{17} or φ_{18}. The same applies to $D(154)^*_{2.22}$. For the simple modules belonging to B_4, we get:

$$D(120)_{2.22} \leftrightarrow \varphi_{14},\ D(210_1)_{2.22} \leftrightarrow \varphi_{19}.$$