$3.M_{22}$ in characteristic 2

- $F[3.M_{22}]$ possesses two faithful blocks of positive defect. These are the blocks B_2 and $B_3 = B_2^*$ of defect 7.

- $P \in \mathrm{Syl}_2(3.M_{22})$

- $N_{3.M_{22}}(P) \cong C_3 \times P$; all simple $FN_{3.M_{22}}(P)$-modules are one-dimensional

- The Brauer correspondent b_2 of B_2 contains a unique simple module which is denoted by 1_2. The Brauer correspondent b_3 of B_3 contains the simple module $1_3 = 1_2^*$.

- All simple modules belonging to B_2 and B_3, respectively, have vertex P. The Loewy structures of their Green correspondents are as follows:

block	B_2	B_3	B_2	B_3	B_2	B_3
module	$D(6)_{3.22}$	$D(6)^*_{3.22}$	$D(15)_{3.22}$	$D(15)^*_{3.22}$	$D(45_1)_{3.22}$	$D(45_1)^*_{3.22}$
Green	6	6	15	15	45	45
layer dims.	$1,1,1,$ $1,1,1$	$1,1,1,$ $1,1,1$	$1,2,2,2,$ $2,2,2,1,1$	$1,2,2,2,$ $2,2,2,1,1$	$2,3,4,6,5,6,$ $5,5,3,3,2,1$	$1,2,3,4,5,6,$ $6,6,5,4,2,1$

block	B_2	B_3	B_2	B_3
module	$D(45_2)_{3.22}$	$D(45_2)^*_{3.22}$	$D(84)_{3.22}$	$D(84)^*_{3.22}$
Green	45	45	84	84
layer dims.	$1,2,3,4,5,6,$ $6,6,5,4,2,1$	$2,3,4,6,5,6,$ $5,5,3,3,2,1$	$2,4,6,8,9,10,10,$ $10,8,6,5,3,2,1$	$2,4,6,8,9,10,10,$ $10,8,6,5,3,2,1$

- For each simple module belonging to B_2 or B_3, the Loewy layers of its sources and those of its Green correspondent coincide.

- $3.M_{22} \longrightarrow M_{22}$ the natural epimorphism; a and b standard generators of M_{22}

- standard generators of $3.M_{22}$: preimages A of a, B of b where A has order 2 and B has order 4

- representative for conjugacy class $7A$: $(ABABABABBBABB)^{36}$

- representative for conjugacy class $11B$: $(AB)^6$

module	conj. class	modular char. value	Brauer char.
$D(6)^*_{3.22}$	$11B$	$\mathbb{F}_4.1^2 = \overline{-b11**}$	$\overline{\varphi_8}$
$D(45_1)^*_{3.22}$	$7A$	$0 = \overline{b7}$	$\overline{\varphi_{11}}$

Hence:

$B_2 : D(6)_{3.22} \leftrightarrow \varphi_8,\ D(15)_{3.22} \leftrightarrow \varphi_9,\ D(45_1)_{3.22} \leftrightarrow \varphi_{11},\ D(45_2)_{3.22} \leftrightarrow \varphi_{10},\ D(84)_{3.22} \leftrightarrow \varphi_{12};$

$B_3 : D(6)^*_{3.22} \leftrightarrow \overline{\varphi_8},\ D(15)^*_{3.22} \leftrightarrow \overline{\varphi_9},\ D(45_1)^*_{3.22} \leftrightarrow \overline{\varphi_{11}},\ D(45_2)^*_{3.22} \leftrightarrow \overline{\varphi_{10}},\ D(84)^*_{3.22} \leftrightarrow \overline{\varphi_{12}}.$