$4.M_{22}$ in characteristic 3

- $F[4.M_{22}]$ has 15 blocks; the faithful ones of positive defect are the blocks B_5 and $B_6 = B_5^*$ of defect 2 with elementary abelian defect groups.
- $P \in \mathrm{Syl}_3(4.M_{22})$; $N_{4.M_{22}}(P)$ is isomorphic to a split extension $P{:}(C_4{:}C_8)$.
- The Brauer correspondent of B_5 is denoted by b_5, and it contains the simple modules 1_2, 1_3, 1_4, 1_5, 2_1. The Brauer correspondent of B_6 is denoted by b_6, and it contains the simple modules $1_6 = 1_2^*$, $1_7 = 1_3^*$, $1_8 = 1_4^*$, $1_9 = 1_5^*$, $2_2 = 2_1^*$.
- All simple $F[4.M_{22}]$-modules belonging to B_5 and B_6, respectively, have vertex P. For a suitable labelling, their Green correspondents have the following Loewy structures:

block	B_5				
module	$D(56_1)_{4.22}$	$D(56_2)_{4.22}$	$D(64)_{4.22}$	$D(160_1)_{4.22}$	$D(160_2)_{4.22}$
Green	$\begin{bmatrix} 1_2\ 2_1 \\ 1_3\ 1_4\ 1_5 \\ 2_1\ 2_1 \\ 1_2 \end{bmatrix}$	$\begin{bmatrix} 1_3\ 2_1 \\ 1_3\ 2_1 \\ 1_2\ 1_4\ 1_5 \\ 2_1 \end{bmatrix}$	$\begin{bmatrix} 1_2\ 1_4\ 1_5 \\ 2_1\ 2_1 \\ 1_3\ 1_4\ 1_5 \end{bmatrix}$	$\begin{bmatrix} 1_4\ 2_1 \\ 1_2\ 1_3\ 1_5\ 2_1 \\ 1_2\ 1_3\ 1_5\ 2_1 \\ 1_4\ 2_1 \end{bmatrix}$	$\begin{bmatrix} 1_5\ 2_1 \\ 1_2\ 1_3\ 1_4\ 2_1 \\ 1_2\ 1_3\ 1_4\ 2_1 \\ 1_5\ 2_1 \end{bmatrix}$

block	B_6				
module	$D(56_1)_{4.22}^*$	$D(56_2)_{4.22}^*$	$D(64)_{4.22}^*$	$D(160_1)_{4.22}^*$	$D(160_2)_{4.22}^*$
Green	$\begin{bmatrix} 1_2^*\ 2_1^* \\ 1_2^*\ 2_1^* \\ 1_3^*\ 1_4^*\ 1_5^* \\ 2_1^* \end{bmatrix}$	$\begin{bmatrix} 1_3^*\ 2_1^* \\ 1_2^*\ 1_4^*\ 1_5^* \\ 2_1^*\ 2_1^* \\ 1_3^* \end{bmatrix}$	$\begin{bmatrix} 1_3^*\ 1_4^*\ 1_5^* \\ 2_1^*\ 2_1^* \\ 1_2^*\ 1_4^*\ 1_5^* \end{bmatrix}$	$\begin{bmatrix} 1_4^*\ 2_1^* \\ 1_2^*\ 1_3^*\ 1_5^*\ 2_1^* \\ 1_2^*\ 1_3^*\ 1_5^*\ 2_1^* \\ 1_4^*\ 2_1^* \end{bmatrix}$	$\begin{bmatrix} 1_5^*\ 2_1^* \\ 1_2^*\ 1_3^*\ 1_4^*\ 2_1^* \\ 1_2^*\ 1_3^*\ 1_4^*\ 2_1^* \\ 1_5^*\ 2_1^* \end{bmatrix}$

- If D is a simple module belonging to B_5 or B_6 then the restriction L of its Green correspondent $f(D)$ in $N_{4.M_{22}}(P)$ to P is a source of D. Moreover, L and $f(D)$ have the same Loewy lengths, and the dimensions of their Loewy layers coincide.

- $4.M_{22} \longrightarrow M_{22}$ the natural epimorphism, a and b standard generators of M_{22}
- standard generators of $4.M_{22}$: preimages A of a and B of b where A has order 2, AB has order 11 and $ABABB$ has order 11

- $g := ABABABBBABB$ is an element of order 8
- $z := (ABABABABABBABABBABB)^{63} \in Z(4.M_{22})$ has order 4
- representative for conjugacy class $11A$: AB
- $\zeta := \exp(2\pi i/8)$, $^{-} : \mathbb{Z}[\zeta] \longrightarrow \mathbb{F}_9$, $\zeta \longmapsto \mathbb{F}_9.1$ the lifting map

The central element z acts on B_5 via multiplication with $\mathbb{F}_9.1^2$, and on B_6 via multiplication with $\mathbb{F}_9.1^6$. Moreover, we have

module	conj. class	modular char. value
$D(56_2)_{4.22}$	g	$\mathbb{F}_9.1 = \overline{-2z_8}$
$D(56_1)_{4.22}$	g	$2\mathbb{F}_9.1 = \overline{2z_8}$
$D(160_1)_{4.22}$	$11A$	$1 = \overline{-b11}$