$M_{22}\!:\!2$ in characteristic 3

- $F[M_{22}:2]$ has nine blocks; the ones of positive defect are the principal block B_1 with elementary abelian defect groups of order 9, and the blocks B_2 and B_3 of defect 1.
- $P \in \mathrm{Syl}_3(M_{22})$; $N_{M_{22}:2}(P)$ is isomorphic to a split extension $N_{M_{22}}(P)\!:\!2 \cong M_9\!:\!2$.
- The simple $FN_{M_{22}:2}(P)$-modules are denoted by $1_1 = F$, 1_2, 1_3, 1_4, 2_1, 2_2, $2_3 = 2_2^*$, according to their dimensions.
- $C \cong C_3$ denotes a common defect group of the blocks B_2 and B_3; $N := N_{M_{22}:2}(C)$ has order 144, and $N_{M_{22}:2}(C)/C \cong \mathfrak{S}_4 \times C_2$. We identify $N_{M_{22}:2}(C)/C$ with $\mathfrak{S}_4 \times C_2$ and view it as the inner direct product of the subgroups $H_1 := \{(x,1) \mid x \in \mathfrak{S}_4\}$ and C_2.
- The simple $F[M_{22}\!:\!2]$-modules belonging to the blocks of positive defect have the defect groups of their blocks as vertices. For a suitable labelling, the Loewy structures of their Green correspondents and those of their sources are as follows:

block	B_1						
mod.	$D(1_1)_{22:2}$	$D(1_2)_{22:2}$	$D(55_1)_{22:2}$	$D(55_2)_{22:2}$	$D(98)_{22:2}$	$D(231_1)_{22:2}$	$D(231_2)_{22:2}$
Green	F	1_3	1_2	1_4	$\begin{bmatrix} 2_1 \\ 2_2\ 2_3 \\ 2_1 \end{bmatrix}$	$\begin{bmatrix} 2_2 \\ 1_1\ 1_4 \\ 2_3 \end{bmatrix}$	$\begin{bmatrix} 2_3 \\ 1_2\ 1_3 \\ 2_2 \end{bmatrix}$
source	F	F	F	F	$\begin{bmatrix} F \\ F\ F \\ F \end{bmatrix}$	$\begin{bmatrix} F \\ F \\ F \end{bmatrix}$	$\begin{bmatrix} F \\ F \\ F \end{bmatrix}$

block	B_2		B_3	
module	$D(21_1)_{22:2}$	$D(210_1)_{22:2}$	$D(21_2)_{22:2}$	$D(210_2)_{22:2}$
Green	$\mathrm{Inf}_C^N(D^{(3,1)} \boxtimes F)$	$\mathrm{Inf}_C^N(D^{(2,1^2)} \boxtimes \mathbf{sgn})$	$\mathrm{Inf}_C^N(D^{(3,1)} \boxtimes \mathbf{sgn})$	$\mathrm{Inf}_C^N(D^{(2,1^2)} \boxtimes F)$
source	F	F	F	F

- standard generators of $M_{22}\!:\!2$: $c := (3,18)(4,16)(5,6)(7,11)(8,17)(12,15)(13,21)$, $d := (1,2,13,8)(3,9,16,10)(4,17,12,7)(5,20)(6,14,22,11)(15,18)(19,21)$
- representative for conjugacy class $2B$: $(cdd)^3$
- representative for conjugacy class $2C$: $(cdcdd)^5$

module	conj. class	modular char. value	Brauer char.
$D(55_1)_{22:2}$	$2B$	1	$\varphi_{7,0}$
$D(231_1)_{22:2}$	$2C$	1	$\varphi_{10,0}$
$D(21_1)_{22:2}$	$2B$	1	$\varphi_{2,0}$
$D(210_2)_{22:2}$	$2B$	2	$\varphi_{9,0}$

Hence, for the simple modules belonging to B_1, we get:

$$D(1_1)_{22:2} = F \leftrightarrow \varphi_{1,0},\ D(1_2)_{22:2} \leftrightarrow \varphi_{1,1}, \quad D(55_1)_{22:2} \leftrightarrow \varphi_{7,0}, \quad D(55_2)_{22:2} \leftrightarrow \varphi_{7,1},$$
$$D(98)_{22:2} \leftrightarrow \varphi_5, \quad D(231_1)_{22:2} \leftrightarrow \varphi_{10,0},\ D(231_2)_{22:2} \leftrightarrow \varphi_{10,1}\ ,$$

and for the simple modules belonging to B_2 and B_3, respectively, we get:

$$B_2 : D(21_1)_{22:2} \leftrightarrow \varphi_{2,0},\ D(210_1)_{22:2} \leftrightarrow \varphi_{9,1}, \quad B_3 : D(21_2)_{22:2} \leftrightarrow \varphi_{2,1},\ D(210_2)_{22:2} \leftrightarrow \varphi_{9,0}.$$