

M_{22} in characteristic 2

- FM_{22} has only one block, i.e. the principal block B_1 of defect 7.
- $Q \in \text{Syl}_2(M_{22})$; $N_{M_{22}}(Q) = Q$
- All simple FM_{22} -modules have vertex Q . The Loewy structures of their Green correspondents are as follows:

module	$D(1)_{22}$	$D(10)_{22}$	$D(10)_{22}^*$	$D(34)_{22}$	$D(70)_{22}$
Green	1	10	10	34	70
layer dims.	1	1, 2, 1, 2, 1, 1, 1, 1	1, 1, 1, 2 2, 1, 1, 1	2, 3, 3, 5, 4, 5, 4, 4, 2, 2	1, 3, 4, 6, 7, 8, 9, 8, 7, 6, 5, 3, 2, 1

module	$D(70)_{22}^*$	$D(98)_{22}$
Green	70	98
layer dims.	1, 3, 4, 6, 7, 8, 9, 8, 7, 6, 5, 3, 2, 1	2, 5, 6, 9, 10, 12, 12, 12, 10, 8, 6, 3, 2, 1

- standard generators for M_{22} : $a := (1, 10)(3, 18)(4, 8)(5, 12)(6, 15)(13, 21)(14, 22)(16, 17)$,
 $b := (1, 8, 18, 7)(2, 10)(3, 13, 20, 6)(4, 16, 14, 9)(5, 11, 17, 21)(12, 19)$
- representative for conjugacy class 7A: $sA := ababababbbabb$
- representative for conjugacy class 11A: $eA := ab$

module	conj. class	modular char. value	Brauer char.
$D(10)_{22}$	7A	0	φ_2
$D(70)_{22}$	11A	$\mathbb{F}_4.1^2$	φ_5

Hence:

$$\begin{aligned}
 D(1)_{22} = F &\leftrightarrow \varphi_1, & D(10)_{22} &\leftrightarrow \varphi_2, & D(10)_{22}^* &\leftrightarrow \varphi_3, & D(34)_{22} &\leftrightarrow \varphi_4, \\
 D(70)_{22} &\leftrightarrow \varphi_5, & D(70)_{22}^* &\leftrightarrow \varphi_6, & D(98)_{22} &\leftrightarrow \varphi_7.
 \end{aligned}$$