M_{22} in characteristic 3

- FM_{22} has five blocks: the principal block B_1 with elementary abelian defect groups of order 9, the block B_2 of defect 1, and the blocks B_3 , $B_4 = B_3^*$ and B_5 of defect 0.
- $P \in \text{Syl}_3(M_{22}); N_{M_{22}}(P) \cong M_9$ has order 72.
- The simple $FN_{M_{22}}(P)$ -modules are denoted by $1_1 = F, 1_2, 1_3, 1_4, 2$. The modules $1_2, 1_3, 1_4$ are permuted by outer automorphisms of $FN_{M_{22}}(P)$.
- Let C be a defect group of B_2 . Then $N_{M_{22}}(C) \cong \mathfrak{S}_4$, and the simple $FN_{M_{22}}(C)$ -modules are denoted by $1_1 = F, 1_2, 3_1, 3_2$. Here $1_2 = \mathrm{Inf}_C^{N_{M_{22}}(C)}(D^{(2^2)}), \ 3_1 = \mathrm{Inf}_C^{N_{M_{22}}(C)}(D^{(3,1)}), \ 3_2 = \mathrm{Inf}_C^{N_{M_{22}}(C)}(D^{(2,1^2)})$.
- All simple FM_{22} -modules have the defect groups of their blocks as vertices. The Loewy series of their Green correspondents and their sources are as follows:

block	B_1				B_2		
module	$D(1)_{22}$	$D(49)_{22}$	$D(49^*)_{22}$	$D(55)_{22}$	$D(231)_{22}$	$D(21)_{22}$	$D(210)_{22}$
Green	F	$\begin{bmatrix} 1_3 \\ 2 \\ 1_2 \end{bmatrix}$	$\begin{bmatrix} 1_2 \\ 2 \\ 1_3 \end{bmatrix}$	1_4	$\begin{bmatrix} 2\\1_4 & 1_1\\2 \end{bmatrix}$	31	3_2
source	F	$\begin{bmatrix} F \\ F F \\ F \end{bmatrix}$	$\begin{bmatrix} F \\ F & F \\ F \end{bmatrix}$	F	$\begin{bmatrix} F \\ F \\ F \\ F \end{bmatrix}$	F	F

- standard generators for M_{22} : a := (1, 10)(3, 18)(4, 8)(5, 12)(6, 15)(13, 21)(14, 22)(16, 17), b := (1, 8, 18, 7)(2, 10)(3, 13, 20, 6)(4, 16, 14, 9)(5, 11, 17, 21)(12, 19)
- representative for conjugacy class 11A: ab

module	conj. class	modular char. value	Brauer char.
$D(49)_{22}$	11A	$2 = \overline{b11}$	$arphi_5$

Hence:

$$B_1: D(1)_{22} = F \leftrightarrow \varphi_1, \ D(49)_{22} \leftrightarrow \varphi_5, \ D(49)^*_{22} \leftrightarrow \varphi_6, \ D(55)_{22} \leftrightarrow \varphi_7, \ D(231)_{22} \leftrightarrow \varphi_{10}, \\ B_2: D(21)_{22} \leftrightarrow \varphi_2, \ D(210)_{22} \leftrightarrow \varphi_9$$