

M_{23} in characteristic 3

- FM_{23} has seven blocks: the principal block B_1 of defect 2 with elementary abelian defect groups, the block B_4 of defect 1, and the blocks $B_2, B_3 = B_2^*, B_5, B_6 = B_5^*$ and B_7 of defect 0.
- $P \in \text{Syl}_3(M_{23})$; $N_{M_{23}}(P)$ is isomorphic to a split extension $M_9:2$
- The simple $FN_{M_{23}}(P)$ -modules are denoted by $1_1 = F, 1_2, 1_3, 1_4, 2_1, 2_2, 2_3 = 2_2^*$, according to their dimensions.
- $C \cong C_3$ denotes a defect group of the block B_4 .
- The normalizer $N_{M_{23}}(C)$ is isomorphic to a split extension $(\mathfrak{A}_5 \times C_3):2$, and $N_{M_{23}}(C)/C \cong \mathfrak{S}_5$. The unique projective simple $F\mathfrak{S}_5$ -module is $D^{(3,1^2)}$ of dimension 6.
- All simple FM_{23} -modules have the defect groups of their blocks as vertices. For a suitable labelling, the Green correspondents and the sources of the simple modules belonging to B_1 and B_4 , respectively, have the following Loewy structures:

block	B_1						
module	$D(1)_{23}$	$D(22)_{23}$	$D(104)_{23}$	$D(104)_{23}^*$	$D(253)_{23}$	$D(770)_{23}$	$D(770)_{23}^*$
Green	F	1_3	$\begin{bmatrix} 1_4 \\ 2_3 \\ 2_1 \end{bmatrix}$	$\begin{bmatrix} 2_1 \\ 2_2 \\ 1_4 \end{bmatrix}$	1_2	2_2	2_3
source	F	F	$\begin{bmatrix} F \\ F \ F \\ F \ F \end{bmatrix}$	$\begin{bmatrix} F \ F \\ F \ F \\ F \end{bmatrix}$	F	F	F

block	B_4
module	$D(231)_{23}$
Green	$\text{Inf}_C^{N_{M_{23}}(C)}(D^{(3,1^2)})$
source	F

- standard generators of M_{23} : $a := (1, 19)(2, 23)(3, 15)(4, 5)(8, 16)(9, 18)(12, 17)(20, 22)$,
 $b := (1, 7, 16, 14)(2, 4, 6, 19)(3, 17, 13, 23)(5, 21)(9, 20)(10, 12, 18, 11)$
- representative for conjugacy class 11A: $ababababbabababb$
- representative for conjugacy class 23A: $(ab)^5$

module	conj. class	modular char. value	Brauer char.
$D(104)_{23}$	$11A$	0	φ_6
$D(770)_{23}$	$23A$	0	φ_{10}

Hence, for the simple modules belonging to B_1 we get:

$$\begin{aligned}
D(1)_{23} = F &\leftrightarrow \varphi_1, & D(22)_{23} &\leftrightarrow \varphi_2, & D(104)_{23} &\leftrightarrow \varphi_6, & D(104)_{23}^* &\leftrightarrow \varphi_5, \\
D(253)_{23} &\leftrightarrow \varphi_8, & D(770)_{23} &\leftrightarrow \varphi_{10}, & D(770)_{23}^* &\leftrightarrow \varphi_9,
\end{aligned}$$

and the simple module $D(231)_{23}$ belonging to B_4 has Brauer character φ_7 .