M_{24} in characteristic 3

- FM_{24} has six blocks: the principal block B_1 of defect 3 with extraspecial defect groups of exponent 3, the blocks B_2, $B_3 = B_2^*$, B_4, B_5 of defect 1, and B_6 of defect 0.

- $P \in \mathrm{Syl}_3(M_{24})$; $N_{M_{24}}(P)$ has order 216, and $N_{M_{24}}(P)/P$ is isomorphic to the dihedral group of order 8. The simple $FN_{M_{24}}(P)$-modules are denoted by $1_1 = F$, 1_2, 1_3, 1_4, 2, according to their dimensions.

- $C_{3,1}$ denotes a common defect group of the blocks B_2, B_3 and B_5. Then $N_{M_{24}}(C_{3,1}) \cong L_2(7) \times \mathfrak{S}_3$. Identifying both groups via this isomorphism, the simple $FN_{M_{24}}(C_{3,1})$-modules can be denoted as $F = 1_1 \boxtimes F$, $F \boxtimes \mathbf{sgn}$, $3_1 \boxtimes F$, $3_2 \boxtimes F$, $3_1 \boxtimes \mathbf{sgn}$, $3_2 \boxtimes \mathbf{sgn}$, $6_1 \boxtimes F$, $6_1 \boxtimes \mathbf{sgn}$, $7_1 \boxtimes F$, $7_1 \boxtimes \mathbf{sgn}$. Here, $\mathbf{sgn}$ denotes the alternating $F\mathfrak{S}_3$-module $D^{(2,1)}$.

- $C_{3,2}$ denotes a defect group of the block B_4; $N_{M_{24}}(C_{3,2})$ is isomorphic to an extension $3.\mathfrak{S}_6$. There are two projective simple $FN_{M_{24}}(C_{3,2})$-modules, namely the inflations of the simple $F\mathfrak{S}_6$-modules $D^{(4,2)}$ and $D^{(2^2,1^2)} = D^{(4,2)} \otimes \mathbf{sgn}$ both of which have dimension 9.

- Apart from the simple module $D(483)_{24}$ belonging to B_1, all simple FM_{24}-modules have the defect groups of their blocks as vertices.

- The module $D(483)_{24}$ has a vertex Q whose normalizer $N_{M_{24}}(Q)$ is isomorphic to $\mathrm{Aut}(M_9)$, that is to a split extension $M_9 : \mathfrak{S}_3$. Moreover, $N_{M_{24}}(Q)/Q \cong \mathrm{GL}(2,3)$ which is one of the double covers of $\mathfrak{S}_4$. There are two projective simple $F[N_{M_{24}}(Q)/Q]$-modules, namely the inflation 3_1 of the simple $F\mathfrak{S}_4$-module $D^{(3,1)}$, and the inflation 3_2 of the simple $F\mathfrak{S}_4$-module $D^{(2,1^2)}$. Both of these have dimension 3.

- For a suitable labelling, the Green correspondents and the sources of the simple FM_{24}-modules belonging to the blocks of positive defect have the following Loewy structures:

block	B_1						
module	$D(1)_{24}$	$D(22)_{24}$	$D(231)_{24}$	$D(770)_{24}$	$D(770)_{24}^*$	$D(1243)_{24}$	$D(483)$
Green	F	1_2 2 1_2	2 $1_1\ 1_3$ 2	1_4 2 $1_3\ 1_4$	$1_3\ 1_4$ 2 1_4	2 $1_1\ 1_2\ 1_3$ $2\ 2$ $1_3\ 1_4$ $2\ 2$ $1_1\ 1_2$ 2	3_1
source layer dims.	1	$1,2,1$	$1,1,1$	$1,2,2$	$2,2,1$	$2,3,4,2,4,2,2$	1

block	B_2		B_3		B_5	
module	$D(45)_{24}$	$D(990)_{24}$	$D(45)^*_{24}$	$D(990)^*_{24}$	$D(1035)_{24}$	$D(2277)_{24}$
Green	$3_1 \boxtimes \mathbf{sgn}$	$3_1 \boxtimes F$	$3_2 \boxtimes \mathbf{sgn}$	$3_2 \boxtimes F$	$6_1 \boxtimes F$	$6_1 \boxtimes \mathbf{sgn}$
source	F	F	F	F	F	F

block	B_4	
module	$D(252)_{24}$	$D(5544)_{24}$
Green	$\mathrm{Inf}_{C_{3,2}}^{N_{M_{24}}(C_{3,2})}(D^{(4,2)})$	$\mathrm{Inf}_{C_{3,2}}^{N_{M_{24}}(C_{3,2})}(D^{(2^2,1^2)})$
source	F	F

- standard generators of M_{24}:
  $a := (1,13)(2,20)(3,8)(4,17)(5,14)(6,7)(9,22)(10,18)(11,19)(12,23)(15,24)(16,21)$, $b := (2,13,10)(3,4,14)(5,7,18)(6,19,15)(9,11,22)(17,21,23)$
- representative for conjugacy class $7A$: $(ababababbabbabababababbabb)^3$
- representative for conjugacy class $23A$: ab

module	conj. class	modular char. value	Brauer char.
$D(770)_{24}$	$23A$	0	φ_9
$D(45)_{24}$	$7A$	$\mathbb{F}_9.1^5 = 2\mathbb{F}_9.1 = \overline{b7}$	φ_3

Hence, for the simple modules belonging to B_1 we get:

$$\begin{array}{llll} D(1)_{24} = F \leftrightarrow \varphi_1, & D(22)_{24} \leftrightarrow \varphi_2, & D(231)_{24} \leftrightarrow \varphi_5, & D(483)_{24} \leftrightarrow \varphi_7, \\ D(770)_{24} \leftrightarrow \varphi_9, & D(770)^*_{24} \leftrightarrow \varphi_8, & D(1243)_{24} \leftrightarrow \varphi_{13}, & \end{array}$$

and for the simple modules belonging to the remaining blocks of positive defect we get:

B_2	B_3	B_5	B_4
$D(45)_{24} \leftrightarrow \varphi_3$	$D(45)^*_{24} \leftrightarrow \varphi_4$	$D(1035)_{24} \leftrightarrow \varphi_{12}$	$D(252)_{24} \leftrightarrow \varphi_6$
$D(990)_{24} \leftrightarrow \varphi_{10}$	$D(990)^*_{24} \leftrightarrow \varphi_{11}$	$D(2277)_{24} \leftrightarrow \varphi_{14}$	$D(5544)_{24} \leftrightarrow \varphi_{15}$