

**WORKSHOP ON TOPOLOGICAL DYNAMICS
AND ROTATION THEORY ON SURFACES
ABSTRACTS**

Salvador Addas-Zanata (Universidade de Sao Paulo, Brasil))

Title: *A condition that implies full homotopical complexity of orbits*

Abstract: We studied homeomorphisms $f : S \rightarrow S$ homotopic to the identity, where S is a compact oriented surface of genus $g > 1$, without boundary, satisfying the following hypothesis.

There exists a set of finitely many f -periodic orbits, such that their trajectories under the isotopy of f satisfy the following:

1) If we consider the geodesics in S which are freely homotopic to each trajectory, then the complement of their union is a union of open topological disks.

2) Moreover, we assume that for each of the above geodesics, there are two periodic orbits whose trajectories under the isotopy of f are freely homotopic to that geodesic, one for each orientation.

Then the following main results are proved:

Theorem 1: *If $\tilde{f} : D \rightarrow D$ is the lift of f to the Poincaré disk D which commutes with all the Deck transformations, given any Deck transformation g , there exists a point $\tilde{x} \in D$ and an integer $n > 0$ such that $\tilde{f}^n(\tilde{x}) = g.\tilde{x}$ (this is like saying that the "homotopical rotation set" has zero in the interior, whatever this means...)*

Theorem 2: *If f is a $C^{1+\epsilon}$ diffeomorphism, then there exists a \tilde{f} -periodic point $\tilde{P} \in D$, which is a hyperbolic saddle, such that its unstable manifold has a topologically transverse intersection with the stable manifold of $g.\tilde{P}$, for all deck transformations g .*

This second result is a generalization to surfaces of higher genus of a theorem originally proved in the torus [1]. It has many interesting consequences, particularly in the area preserving case.

Coauthor: Bruno de Paula Jacoia

Jan P. Boronski (AGH Krakow & IT4Innovations Ostrava)

Title: *A compact minimal space whose Cartesian square is not minimal*

Abstract: In my talk I shall outline a construction of a family of 1-dimensional minimal spaces from [1], whose existence answer the following problem in the negative.

Problem. Is minimality preserved under Cartesian product in the class of compact spaces?

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Note that for the fixed point property this question had been resolved in the negative already 50 years ago by Lopez [3], and similar counterexamples do not exist for flows, as shown by Dirbák [2].

References

[1] Boronski J.P.; Clark A.; Oprocha P., A compact minimal space Y such that its square $Y \times Y$ is not minimal. arXiv:1612.09179

[2] Dirbák, M. Minimal extensions of flows with amenable acting groups. Israel J. Math. 207 (2015), no. 2, 581-615.

[3] Lopez, W. An example in the fixed point theory of polyhedra. Bull. Amer. Math. Soc. 73 1967 922-924.

Coauthors: Alex Clark and Piotr Oprocha

Jernej Cinc (University of Vienna, Austria)

Title: *Planar embeddings of inverse limit spaces of tent maps*

Abstract: In this talk I will discuss construction of planar embeddings of inverse limit spaces of tent maps (natural extensions of tent maps) X and the (non)-extendability of the natural shift self-homeomorphism on X to the whole plane.

Sylvain Crovisier (Université Paris-Sud, France)

Title: *Dynamics of strongly dissipative diffeomorphisms of the disc with zero entropy*

Abstract: We consider the diffeomorphisms of the disc satisfying a dissipation assumption. (It includes the dynamics of Hénon diffeomorphisms with Jacobian $< 1/4$.) We describe the dynamics of these systems when the topological entropy vanishes, in particular the structure of the periodic set and its renormalizations.

Coauthors: Enrique Pujals and Charles Tresser.

Pierre-Antoine Guihéneuf (Université Pierre et Marie Curie, France)

Title: *Discretizations of dynamics and basic applications to the numerical approximation of rotation sets*

Abstract: In this talk I will begin by presenting some results about discretizations of generic dynamics. I will concentrate on the phenomenon of stabilization of periodic orbits that gives a "bad algorithm" (but still better than the naive one) to compute the rotation set of a torus homeo.

Tobias Jäger (Universität Jena, Germany)

Title: *Linearisation in surface dynamics*

Abstract: Poincaré's seminal result on orientation-preserving circle homeomorphisms guarantees the existence of a semi-conjugacy to an irrational rotation whenever the rotation number is irrational. We discuss a general scheme that allows to extend this result to more general settings in surface dynamics. This allows to characterise, in particular, under which conditions irrational rotations appear as factors of irrational torus homeomorphisms and as factors for the dynamics on invariant cofrontiers.

Coauthors: Andres Koropecki, Fabio A. Tal

Alejandro Kocsard (Universidade Federal Fluminense, Brasil)

Title: *Wandering domains and irrational factors for periodic point free homeomorphisms*

Abstract: In this talk we shall discuss the problem of whether a periodic point free torus homeomorphism admits an irrational circle rotation as a topological factor, and how the geometry of the set of wandering points appears as an obstruction for it.

Andres Koropecki (Universidade Federal Fluminense, Brasil / Universität Jena, Germany)

Title: *Rotational behavior for a standard family on the torus*

Abstract: We consider a two-parameter family of diffeomorphisms which is the composition of a horizontal and a vertical skew-rotations in the torus: $F_{\alpha,\beta} = h_{\alpha} \circ v_{\beta}$ where

$$h_{\alpha}(x, y) = (x + \alpha \sin(2\pi y), y) \text{ and } v_{\beta}(x, y) = (x, y + \beta \sin(2\pi x)).$$

This is probably the simplest kind of explicit example where one has interesting rotational behavior. We are interested in understanding the rotation behavior for arbitrary parameters, and in particular the transition between empty and nonempty interior in the rotation set. This type of question which is related to the variation of rotation sets after perturbations is very poorly understood in general, and this "standard" family already raises many intriguing questions.

I will talk about some partial results in this subject, which also apply to other families of the same kind.

Coauthors: Tobias Jäger, Fabio A. Tal.

Frédéric Le Roux (Université Pierre et Marie Curie, France)

Title: *Polynomial entropy of Brouwer homeomorphisms*

Abstract: Polynomial entropy is a dynamical invariant which is relevant for zero entropy systems, in particular when the non-wandering set is finite. We will see that for Brouwer homeomorphisms, the polynomial entropy can be either 1 (for the translation) or any value greater than or equal to 2.

Coauthor: Louis Hauseux.

Xiaoxuan Liu (IMPA, Brazil)

Title: *Rotation vectors for minimal two torus diffeomorphisms*

Abstract: In this talk, we will look at the pointwise rotation vector for a smooth two torus diffeomorphism. We show there exist examples where for Lebesgue almost every point of \mathbb{T}^2 , the pointwise rotation vector is not well defined. That is, when we try to use the usual way to define this number, the limit does not exist. The method we use, is a variant of Artur Avila’s method to obtain a counter-example of Franks-Misiurewicz conjecture. Artur’s method is a variant of Anosov-Katok method of fast approximation and conjugation.

Coauthors: A.Avila and D.Xu

Piotr Oprocha (University of Science and Technology Krakow, Poland)

Title: *On disjointness with all minimal systems and synchronization of return times*

Abstract: In 1967 Furstenberg introduced the notion of topological disjointness and stated the following question: Describe the class of system disjoint with all distal systems and with all minimal systems. He also proved that both classes are nonempty. The first class was very quickly fully characterized in short and elegant paper by Petersen. After 50 years the second problem still remains open, even when restricted to the class of transitive systems. Still, some “upper” and “lower” bounds on the shape of this class are known. The aim of this talk is present history and some recent results on sharpening these bounds.

Katja Polotzek (Max Planck Institute for the Physics of Complex Systems, Dresden, Germany)

Title: *Set-oriented numerical approximation of rotation sets*

Abstract: For homeomorphisms of the two-torus, homotopic to the identity, the rotation set is known to be compact and convex and, generically, it appears polygonal. In general, however, its specific shape is unknown analytically.

In the talk, direct approaches to the numerical approximation of rotation sets are presented roughly in order to illustrate the limits of a pointwise detection. Alternatively, we formulate a set-oriented algorithm and obtain good numerical representations of the rotation sets for several example maps.

By the use of ϵ -pseudo orbits, we define an ϵ -rotation set and prove the convergence of these sets to the original rotation set as the perturbation ϵ decreases to zero. This theorem gives a theoretical basis for our algorithm along with an error estimate.

Coauthors: T. Jäger, K. Padberg-Gehle

Martín Sambarino (Universidad de la República, Uruguay)

Title: *Contributions to the Franks-Misiurewicz conjecture*

Abstract: With A. Koropecki and A. Passeggi we have proved the FM Conjecture (rational slope) in the case of extensions of irrational rotations. In this talk we will extend this result to the case when the bounded deviation property holds by showing that they are indeed extensions of irrational rotations.

Fabio A. Tal (Universidade de São Paulo, Brasil / Universität Jena, Germany)

Title: *Topological horseshoes for surface homeomorphisms*

Abstract: We will present a new criteria, based on a previously developed forcing theory for Brouwer equivariant theory, to identify the existence of topological horseshoes in surface homeomorphisms. Some applications for the description of zero entropy homeomorphisms of surfaces with null genus will be given.

Coauthor: Patrice Le Calvez