Workshop on
"Dynamical Systems and Dimension Theory"

8. - 12. September 2014, Wöltingerode, Germany,
organized by G. Fuhrmann and T. Jäger

Talks & Abstracts

Minicourses (4×90 min)

Gerhard Keller *An elementary proof for the dimension of the graph of the classical Weierstrass function.*

Let $W_{\lambda,b}(x) = \sum_{n=0}^{\infty} \lambda^n g(b^n x)$ where $b \geq 2$ is an integer and $g(u) = \cos(2\pi u)$ (classical Weierstrass function) or $b = 2$ and $g(u) = \mathrm{dist}(u, \mathbb{Z})$. Building on work by Barańsky, Bárány and Romanowska [1] and Tsujii [4], we provide elementary proofs that the Hausdorff dimension of $W_{\lambda,b}$ equals $2 + \frac{\log \lambda}{\log b}$ for all $\lambda \in (\lambda_b, 1)$ with a suitable $\lambda_b < 1$. This reproduces results by Ledrappier [2] and Barańsky, Bárány and Romanowska [1] without using the dimension theory for hyperbolic measures of Ledrappier and Young [3], which is replaced by a simple telescoping argument.
The proof I will present is elementary in the sense that its prerequisits are covered by the usual courses in real analysis including measure theory, a first course in measure theoretic probability and one basic fact from Hilbert spaces (weak compactness of the unit ball). It can be found in the recent paper http://arxiv.org/abs/1406.3571.

[1] K. Barański, B. Bárány, and J. Romanowska. On the dimension of the graph of the classical Weierstrass function. *arXiv:1309.3759* (2013), 1–19.

[2] F. Ledrappier. On the dimension of some graphs. *Contemp. Math* (1992), 135: 285–293.

[3] F. Ledrappier and L.-S. Young. The metric entropy of diffeomorphisms. Part II: Relations between entropy, exponents and dimension. *Ann. of Math.* (1985), 122 (3): 540–574.

[4] M. Tsujii. Fat solenoidal attractors. *Nonlinearity* (2001), 14 (5): 1011–1027.

Daniel Lenz *Dynamical systems and the diffraction of quasicrystals.*

Since their discovery in 1982, quasicrystals have been considered from various points of view in several disciplines. From the mathematical side, the treatment of diffraction and the connection to dynamical systems has received ample attention. We provide an introduction into this line of research.

Extended talks (90 min)

Katrin Gelfert *Entropy spectrum for Lyapunov exponents in non-hyperbolic dynamics*, joint work with M. Rams and L. Díaz.

We describe the topological entropy of level sets for central Lyapunov exponents. Here, we study a class of partially hyperbolic dynamical systems which are genuinely non-hyperbolic in the sense that there exists contracting and expanding behavior in the central direction (porcupines).

Roland Zweimüller *TBA*

Regular talks (45 min)

Kurt Falk *Conformal ending measures on limit sets of Kleinian groups.*

I will present an alternative method of constructing conformal measures on limit sets of Kleinian groups. As opposed to the usual method by Patterson, this new construction relies on the convergence type property of the considered Kleinian group.

Gabriel Fuhrmann *On the geometry of Strange Non-chaotic Attractors*, joint work with M. Gröger and T. Jäger.

This talk deals with quasi-periodically forced monotone interval maps of the form

$$f\colon \mathbb{T}\times\mathbb{R}\to\mathbb{T}\times\mathbb{R}, \qquad (\theta,x)\mapsto(\theta+\omega,g(\theta,x))$$

where $\mathbb{T}:=\mathbb{R}/\mathbb{Z}$, ω is Diophantine and $g(\theta,\cdot)$ is monotonously increasing and concave for each $\theta\in\mathbb{T}^d$. Under some mild assumptions, bifurcations of such systems yield "strange" minimal sets.
We answer a question by Herman on the topological structure of these minimal sets, basically showing that they don't have gaps. A similar result has earlier been derived by Bjerklöv for the special case of a projective dynamical system associated to a quasi-periodic Schrödinger cocycle [1]. We provide a simplified proof of an extension of his result to systems of the above form.
Further, we compute the Hausdorff dimension of the upper bounding graphs of these minimal sets.

[1] K. Bjerklöv. Dynamics of the Quasi-Periodic Schrödinger Cocycle at the Lowest Energy in the Spectrum. *Comm. Math. Phys.* (2007), 272 (2): 397–442.

Roman Hric *TBA*

Johannes Jaerisch *Hölder regularity of limit state functions in random complex dynamical systems*, joint work with H. Sumi.

We consider the dynamics of semigroups of rational maps on the Riemann sphere and random complex dynamical systems. Under certain conditions, in the limit stage of a transition operator associated with a random complex dynamical system, a complex analogue of a devil's staircase function appears [1]. In this talk, we employ the multifractal formalism in ergodic theory to investigate the spectrum of the Hölder regularity of these functions. In this way, we obtain a refined gradation between chaos and order in random complex dynamical systems.

[1] H. Sumi. Random complex dynamics and semigroups of holomorphic maps. *Proc. London Math. Soc.* (2011), 102 (1): 50–112.

Marc Keßeböhmer *TBA*

Tony Samuel *On the convergence to equilibrium in α-Farey systems.*

The transfer operator $\widehat{T}$ is an important tool in the study of measure preserving dynamical systems (X,μ,T) where the measure μ has infinite mass. For instance, it allows one to obtain generalisations of Birkhoff's Ergodic Theorem. In these generalisations, one studies the asymptotics of the partial sums $\sum_{k=0}^{n}\widehat{T}^k$. However, it turns out that asymptotics of the operators $\widehat{T}^k$ themselves are considerably more delicate. In this talk we will discuss recent results concerning the asymptotic behaviour of the iterates of the transfer operators for α-Farey maps.